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The Introduction to the Special Issue on "RT(N) = a + bN^{-c}": The Power Law of Learning 25 years later

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This special issue of *Tutorials in Quantitative Methods for Psychology* presents four papers on the Power Law of Learning to celebrate the 25th anniversary of A. Newell and P. Rosenbloom's (1981) seminal paper "Mechanism of Skill Acquisition and the Law of Practice". This introduction highlights the main points of Newell and Rosenbloom's work and then presents the contributors' articles.

In psychology, or any other scientific discipline, to assert the discovery of a law is to make a very bold claim. A law describes a "regular, predictable relationship among empirical variables" (Marx, 1967, p. 7); a relationship that has been observed and verified so many times that it is improbable that it will ever be shown to be false (Stanovich, 2007). Thus, the stringency of these criteria prevents scientists from frivolously proclaiming that they have found a law and, as a group, psychologists have certainly not exaggerated in applying this term to the regularities observed in their research. To intuitively verify this last claim, I invite the reader to name all the laws in psychology that he or she can think off. Answers may include the Weber-Fechner Law, Thorndike's Law of Effect, the Yerkes-Dodson law, and the Gestalt Laws, but the reader will realize that the list is not very long. Teigen (2002) investigated the status of laws in psychology more systematically. Searching PsycLit, he found that the term "law"[1] appeared only 3,093 times in the 1.4 million 20th century journal abstracts surveyed. Moreover, the number of "law" citations relative to the total number of publications has decreased considerably during the last few decades. Teigen concluded:

> Strong psychological regularities are difficult to establish beyond doubt [and] they rarely lend themselves to formulation with the rigor and precision of a mathematical function […]. So even if psychology is still searching for laws, the outcome of this search usually is called something else (p. 116).

Hence, when A. Newell and Rosembloom (1981) proposed the Power Law of Practice, they were clearly going against the grain. In this special issue of *Tutorials in Quantitative Methods for Psychology* (TQMP), we celebrate A. Newell and Rosembloom's boldness and the ideas that they presented in "Mechanism of Skill Acquisition and the Law of Practice".

The main goal of A. Newell and Rosenbloom's (1981) three-part paper was to show that there exists a quantitative relation between the time necessary to perform a task and the amount of practice that holds across domains of learning. Starting with the results of Snoody's (1926) classic work using a mirror-tracing task (which are reanalyzed in this special issue by K. Newell, Mayer-Kress, & Liu), they first showed that the Log-Log Learning Law or the Power Law of Practice fit the data from many areas of research including experiments on perceptual-motor skills, perception, motor behavior, elementary decision, memory, complex routines, and problem solving (see Figure 1). If they had decided to conduct the same kind of literature review today, they would have found similar support for the general applicability of their Power Law because it has since

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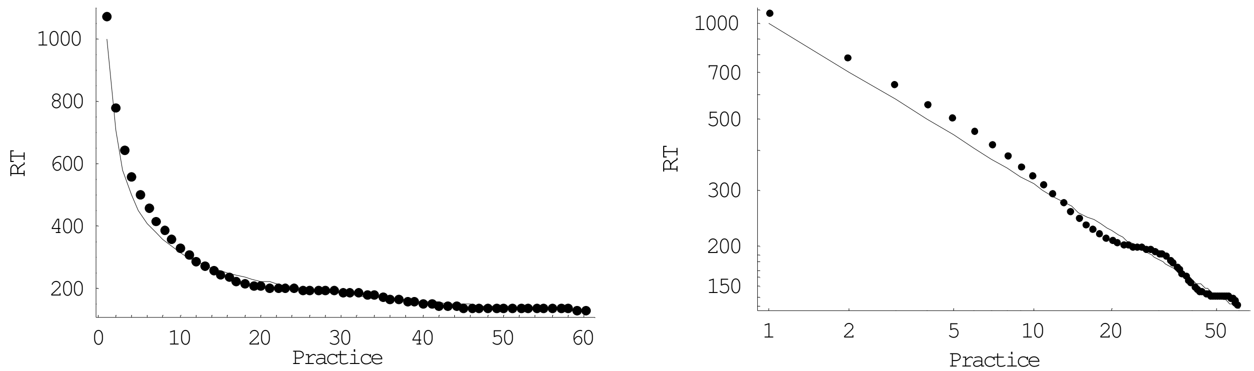

Figure 1. Fictional data fitted to the Power Law of Practice and the Log-Log Learning Law. The power law predicts performance in any given task as a function of the number of training sessions using a power curve for which the general equation is:

$$RT = a + bN^{-c}$$

RT is the time to complete a task at training session N, and a, b, and c are free parameters. They represents respectively the asymptotic response time, the amplitude seen at the beginning of training, and the learning rate. The left panel shows a fictional data set fitted to the Power Law of Practice. If the asymptote is known, then the followings hold true:

$$(RT - a) = bN^{-c}$$

$$Log(RT - a) = -cLog(N) + Log(b)\ .$$

Hence, a graph of RT – a against N, both in log coordinates yields a straight line with a negative slope of –c and an intercept of Log(b). This explains why the Power Law is sometimes called the Log-Log Law. The right panel shows the same fictional data set fitted to the Log-Log Law. Finally, we note that the four-parameter Power Law explored in A. Newell and Rosenbloom (1981) was:

$$RT = a + b(N + e)^{-c}$$

The additional parameter e takes into account the amount of prior experience on the task.

been successfully fitted to data from many different tasks including problem solving (Blessing & Anderson, 1996), lexical decision (Logan, 1988), mental calculation and alphabet-arithmetic (Charness & Campbell, 1988; Logan, 1988; 1992; Woltz, Gardner, & Gyll, 2000), mirror reading (Ofen-Noy, Dudai, Carny, 2003), numerosity judgments (Palmeri, 1997), perceptual classification (Palmeri & Nosofsky, 1997), procedural learning (Woltz, 1988), social judgment (Smith, Branscombe, & Bormann, 1988), speed of processing (Kail, 1991), and search (Chun & Jiang, 2003; Logan, 1998; Strayer & Kramer, 1990).

In the second part of their paper, A. Newell and Rosenbloom (1981) argued that the Power Law fit learning data better than competing families of curves, namely exponentials and hyperbolics. They did so by producing estimates of fit to the data of the experiments reviewed in the first part of their paper. A. Newell and Rosenbloom found that the four parameter Power Law generated fits that were greater or equal to those obtained with the three parameter exponential and hyperbolic functions. This is certainly the portion of their work that has been the most intensely debated and criticized. Heathcote, Brown, and Mewhort (2000) presented what is perhaps the most well-known attack on the Power Law of Practice. In essence, they argued that A. Newell and Rosembloom's fits were highly problematic because they had been applied to averaged data rather than to individual data. When individual data were used, Heathcote et al. found that the exponential function actually provided better fits than the Power Law.

Numerous other arguments and problems have been raised against the Power Law, especially during the last few years. They can be classified into three categories: technical and statistical arguments (e.g. Brown & Heathcote, 2003; Haider & Frensch, 2002), evidence of data that are not well explained by the Power Law (e.g. Kirsner & Speelman, 1996; Rickards, 1997; 1999), and claims that the Power Law over-simplifies the relation between performance and practice because it fails to encompass factors such as transient states (motivation, fatigue, etc.) and participants' prior experiences (e.g. Kirsner & Speelman, in this issue, K. Newell, Mayer-Kress, & Liu, 2001; 2003).

The final part of A. Newell and Rosenbloom's (1981) paper outlined the Chunking Theory of Learning as an explanation of the Power Law of Practice. The theory builds on Miller's (1956) ideas, which confer chunks – meaningful units of knowledge – a fundamental role in understanding information-processing in immediate memory. A. Newell and Rosenbloom speculated that practice on a given task allows people to develop larger and progressively more complex chunks, which in turn, speed up their

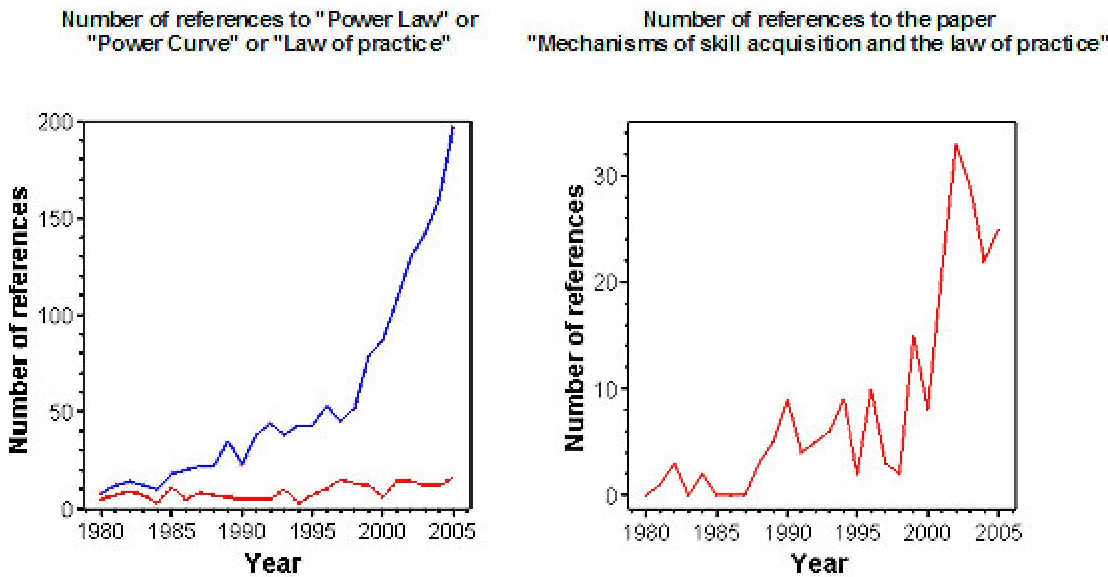

Figure 2. Number of journal articles referring to the power law. The left-hand panel shows the number of abstracts that included the keywords "Power Law", "Power curve", or "Law of Practice" plotted by year of publication. The right-hand panel shows the number of articles that referred to Newell and Rosenbloom (1981) plotted by year of publication. The full lines show the results of searches conducted in the PsycINFO database and the dotted line shows the result of a search conducted in the Medline database. The latter cannot be searched by references cited.

performances. Although A. Newell and Rosenbloom did begin to formalize these ideas in their "Mechanism of Skill Acquisition" paper, it is only later that the Chunking theory led to the development of a complete model. The resulting architecture, known as SOAR (see A. Newell, 1990; Rosenbloom, Laird, & A. Newell, 1993), successfully predicts Power Law driven improvements in human performance. Nevertheless, SOAR is not the only formalized theory to make such a prediction. In fact, many other well-known models also point to the Power Law. This list of models includes other domain-general symbolic architectures such as ACT-R (Anderson, 1992; Anderson & Libiere, 1998), instance-based models (Logan, 1988; 1992; Nosofsky & Palmeri, 1997), and Parallel Distributed Processing networks (Cohen, Dunbar, & McClelland, 1990). Naturally, if it is shown that the Power Law of Practice is not a ubiquitous characterization of the improvements in performances seen with practice, then all of these models stand to answer questions about validity. This is a point that Heathcote et al. (2000) made vigorously.

A. Newell and Rosenbloom's (1981) work has generated a great amount of research. Figure 2 presents the number of research articles that refer to it. It can be observed that there is a clear upward trend, (which by far exceeds the general increase in published research articles[2]). This trend is particularly strong in the human factors and medicine literature.

The papers presented in this special issue serve both to look back on the impact that A. Newell and Rosenbloom's (1981) paper had on the field and to look ahead to the future directions that research on the relation between performance and practice will take. In the first paper, Rosenbloom retraces the personal and intellectual circumstances that led to the development of the ideas he and A. Newell presented in "Mechanism of Skill Acquisition and the Law of Practice". He then describes the work that he and his collaborators conducted during the 80' and 90' to develop different implementations of the Chunking Theory of Learning in an effort to provide an explanation for the Power Law of Practice.

The next two papers present critical analyses of A. Newell and Rosenbloom (1981). Speelman and Kirsner argue that, although the Power Law can satisfactorily parameterize learning data, its main assumption that performance on any given task can be explained as a function of a single variable for the quantity of practice is inadequate. Building on previous work (Speelman, 1995; Speelman & Kirsner, 1993; 2005), they claim that a realistic theory relating learning to practice must incorporate multiple factors including previous practice on any component task making up a new task, the context in which the task is executed, and individual differences.

K. Newell, Mayer-Kress, and Liu dispute the power law's status as a ubiquitous law of learning and reject the notion that changes in performance, which occur with practice, can be understood from a straightforward fit to any function (power law, exponential, or otherwise). Rather, they propose a characteristic time scale model of learning (K. Newell, Liu, & Mayer-Kress, 2001; 2003).

Finally, in keeping with the mission of TQMP to disseminate easy to use introductory guides to quantitative and statistical methods applied to psychological research, we present a tutorial that describes how to use software such Excel, SPSS, and Mathematica to analyze learning data using the Power Law. We also address several methodological and statistical issues related to the analysis of learning curves.

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[1] This number include only usages of "law" when it is refers to a scientific term and excludes other usages. For instance, when it refers to legal issues.

[2] The number of articles indexed in MedLine was 282 199 in 1980 and is 641 995 in 2005, an increase of 230% during the last 25 years; the number of articles indexed in PsychInfo went from 28 040 to 103 889 during the same period, an increase of 370%.