An Introduction to Hierarchical Linear Modeling
Figures 6, 8, 10 and 12 magnified

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This tutorial aims to introduce Hierarchical Linear Modeling (HLM). A simple explanation of HLM is provided that describes when to use this statistical technique and identifies key factors to consider before conducting this analysis. The first section of the tutorial defines HLM, clarifies its purpose, and states its advantages. The second section explains the mathematical theory, equations, and conditions underlying HLM. HLM hypothesis testing is performed in the third section. Finally, the fourth section provides a practical example of running HLM, with which readers can follow along. Throughout this tutorial, emphasis is placed on providing a straightforward overview of the basic principles of HLM.

Specifications for this HLM2 run

Problem Title: no title

The data source for this run = m.test
The command file for this run = C:\DOCUME~1\hlm2.html
Output file name = C:\Documents and Settings\hlm2.html
The maximum number of level-1 units = 300
The maximum number of level-2 units = 30
The maximum number of iterations = 100

Method of estimation: restricted maximum likelihood

The outcome variable is LIFE_SAT

Summary of the model specified

Level-1 Model

$$LIFE\_SAT_{ij} = \beta_{0j} + r_{ij}$$

Level-2 Model

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

Mixed Model

$$LIFE\_SAT_{ij} = \gamma_{00} + u_{0j} + r_{ij}$$

Figure 6 (part 1). HLM output tables – Unconstrained (null) model. This represents a default output in HLM.

Final Results - Iteration 2

Iterations stopped due to small change in likelihood function

$\sigma^2 = 14.60563$

τ
INTRCPT1,β_0 14.95920

Random level-1 coefficient	Reliability estimate
INTRCPT1,β_0	0.911

The value of the log-likelihood function at iteration 2 = -8.621511E+002

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	14.806670	0.739815	20.014	29	<0.001

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	14.806670	0.727380	20.356	29	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	d.f.	χ^2	p-value
INTRCPT1, u_0	3.86771	14.95920	29	326.02031	<0.001
level-1, r	3.82173	14.60563			

Statistics for current covariance components model

Deviance = 1724.302126
Number of estimated parameters = 2

Figure 6 (part 2). HLM output tables – Unconstrained (null) model. This represents a default output in HLM.

Specifications for this HLM2 run

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The maximum number of iterations = 100

Method of estimation: restricted maximum likelihood

The outcome variable is LIFE_SAT

Summary of the model specified

Level-1 Model

$$LIFE\_SAT_{ij} = \beta_{0j} + \beta_{1j} *(SHOTS\_ON_{ij}) + r_{ij}$$

Level-2 Model

$$\beta_{0j} = \gamma_{00} + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + u_{1j}$$

SHOTS_ON has been centered around the group mean.

Mixed Model

$$LIFE\_SAT_{ij} = \gamma_{00}$$
$$+ \gamma_{10} *SHOTS\_ON_{ij} + u_{0j} + u_{1j} *SHOTS\_ON_{ij} + r_{ij}$$

Figure 8 (part 1). HLM output tables – Random intercepts model.

Final Results - Iteration 11

Iterations stopped due to small change in likelihood function

$\sigma^2 = 4.60875$

τ

INTRCPT1,β_0	15.95889	1.39404
SHOTS_ON,β_1	1.39404	0.66199

τ (as correlations)

INTRCPT1,β_0	1.000	0.428
SHOTS_ON,β_1	0.428	1.000

Random level-1 coefficient	Reliability estimate
INTRCPT1,β_0	0.972
SHOTS_ON,β_1	0.539

The value of the log-likelihood function at iteration 11 = -7.182666E+002

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	14.806670	0.739822	20.014	29	<0.001
For SHOTS_ON slope, β_1					
INTRCPT2, γ_{10}	2.889137	0.202235	14.286	29	<0.001

**Final estimation of fixed effects
(with robust standard errors)**

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	14.806670	0.727380	20.356	29	<0.001
For SHOTS_ON slope, β_1					
INTRCPT2, γ_{10}	2.889137	0.198575	14.549	29	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	d.f.	χ^2	p-value
INTRCPT1, u_0	3.99486	15.95889	29	1033.19332	<0.001
SHOTS_ON slope, u_1	0.81547	0.66499	29	64.93933	<0.001
level-1, r	2.14680	4.60875			

Statistics for current covariance components model

Deviance = 1436.533205
Number of estimated parameters = 4

Figure 8 (part 2). HLM output tables – Random intercepts model.

Specifications for this HLM2 run

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Method of estimation: restricted maximum likelihood

The outcome variable is LIFE_SAT

Summary of the model specified

Level-1 Model

$$LIFE\_SAT_{ij} = \beta_{0j} + r_{ij}$$

Level-2 Model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}*(COACH\_EX_j) + u_{0j}$$

COACH_EX has been centered around the grand mean.

Mixed Model

$$LIFE\_SAT_{ij} = \gamma_{00} + \gamma_{01}*COACH\_EX_j + u_{0j} + r_{ij}$$

Figure 10 (part 1). HLM output tables – Means as outcomes model.

Final Results - Iteration 2

Iterations stopped due to small change in likelihood function

$\sigma^2 = 14.60563$

τ
INTRCPT1,β_0 1.67564

Random level-1 coefficient	Reliability estimate
INTRCPT1,β_0	0.534

The value of the log-likelihood function at iteration 2 = -8.384908E+002

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	14.806670	0.323327	45.795	28	<0.001
COACH_EX, γ_{01}	4.784303	0.429936	11.128	28	<0.001

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	14.806670	0.312363	47.402	28	<0.001
COACH_EX, γ_{01}	4.784303	0.373833	12.798	28	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	d.f.	χ^2	p-value
INTRCPT1, u_0	1.29447	1.67564	28	60.12326	<0.001
level-1, r	3.82173	14.60563			

Statistics for current covariance components model

Deviance = 1676.981685
Number of estimated parameters = 2

Figure 10 (part 2). HLM output tables – Means as outcomes model.

Specifications for this HLM2 run

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The maximum number of iterations = 100

Method of estimation: restricted maximum likelihood

The outcome variable is LIFE_SAT

Summary of the model specified

Level-1 Model

$$LIFE\_SAT_{ij} = \beta_{0j} + \beta_{1j}*(SHOTS\_ON_{ij}) + r_{ij}$$

Level-2 Model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}*(COACH\_EX_j) + u_{0j}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11}*(COACH\_EX_j) + u_{1j}$$

SHOTS_ON has been centered around the group mean.
COACH_EX has been centered around the grand mean.

Mixed Model

$$LIFE\_SAT_{ij} = \gamma_{00} + \gamma_{01}*COACH\_EX_j$$
$$+ \gamma_{10}*SHOTS\_ON_{ij} + \gamma_{11}*COACH\_EX_j*SHOTS\_ON_{ij}$$
$$+ u_{0j} + u_{1j}*SHOTS\_ON_{ij} + r_{ij}$$

Figure 12 (part 1). HLM output tables – Random intercepts and slopes model.

Final Results - Iteration 16

Iterations stopped due to small change in likelihood function

$\sigma^2 = 4.60669$

τ
INTRCPT1,β_0 2.67553   0.34947
SHOTS_ON,β_1 0.34947   0.62695

τ (as correlations)
INTRCPT1,β_0 1.000   0.270
SHOTS_ON,β_1 0.270   1.000

Random level-1 coefficient	Reliability estimate
INTRCPT1,β_0	0.853
SHOTS_ON,β_1	0.525

The value of the log-likelihood function at iteration 16 = -6.940699E+002

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	14.806670	0.323326	45.795	28	<0.001
COACH_EX, γ_{01}	4.784303	0.429935	11.128	28	<0.001
For SHOTS_ON slope, β_1					
INTRCPT2, γ_{10}	2.884745	0.199480	14.461	28	<0.001
COACH_EX, γ_{11}	0.381153	0.269730	1.413	28	0.169

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	t-ratio	Approx. d.f.	p-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	14.806670	0.312363	47.402	28	<0.001
COACH_EX, γ_{01}	4.784303	0.373833	12.798	28	<0.001
For SHOTS_ON slope, β_1					
INTRCPT2, γ_{10}	2.884745	0.193475	14.910	28	<0.001
COACH_EX, γ_{11}	0.381153	0.260682	1.462	28	0.155

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	d.f.	χ^2	p-value
INTRCPT1, u_0	1.63571	2.67553	28	190.62228	<0.001
SHOTS_ON slope, u_1	0.79180	0.62695	28	62.86446	<0.001
level-1, r	2.14632	4.60669			

Statistics for current covariance components model

Deviance = 1388.139702
Number of estimated parameters = 4

Figure 12 (part 2). HLM output tables – Random intercepts and slopes model.