2015 ■ Vol. 11 ■ no. 3

DOI: 10.20982/tqmp.11.3.p139

New statistical inference for the Weibull distribution

Xu Zhao[a,✉], Weibu Cheng[a], Yang Zhang[b], Qingnan Zhang[a] & Zhenhai Yang[a]

[a]College of Applied Sciences, Beijing University of Technology, Beijing, 100124, P. R. China
[b]The College of Computer Science, Beijing University of Technology, Beijing, 100124, P. R. China

Abstract ■ Weibull distribution has become a popular tool for modeling life data and improving growth in the field of reliability. The successful application of Weibull distribution to real data depends on the statistical power of hypotheses tests to a large extent. Here we propose two methods to test the shape parameter of a two-parameter Weibull distribution, and its confidence interval is considered. Simulation studies show that coverage of the confidence intervals is close to its desired confidence level, and the two proposed methods exhibited satisfactory performance. A real example employing a Boeing air-conditioning system development study is presented to illustrate the proposed methods.

Keywords ■ Weibull distribution, Gamma distribution, Dirichlet distribution, Rejection region, Power.

✉ zhaox@bjut.edu.cn

Introduction

Demands on electronic products or components (e.g., weapon system, airplane generator, and electronic engines) have become increasingly strict in recent years. Weibull distribution, as one of the most widely used life distribution tools, has been extensively studied in the field of reliability growth research and is often employed to improve data reliability.

Various classical procedures, such as point estimates, hypotheses tests, and confidence intervals, have been proposed for statistical inference. For instance, estimations of unknown parameters, using methods such as the method of moment, the likelihood approach, least square methods, nonlinear regression estimators, robust estimation methods, and Bayesian methods, have been applied to Weibull distribution (Bain & Engelhardt, 1980; Duffy, Starlinger, & Powers, 1993; Lockhart & Stephens, 1994; Kuo & Yang, 1996; Ryan, 2003; Verma & Kapur, 2006; Sürücü & Sazak, 2009). In hypotheses tests, Lockhart and Stephens (1994) introduced the empirical distribution function methods to test whether the considered sample follows the three-parameter Weibull distribution. However, for $n < 10$, such a goodness-of-fit test have insufficient power. Applications of the Weibull distribution to the assessment of hardware or software reliability growth have been extensively investigated by various authors (Bai & Mu, 2011; Ren, Yang, & Meng, 2012).

The remainder of this paper is organized as follows: the first section presents proof of the relationship between Weibull distribution and other common distributions. The subsequent section proposes two original and novel methods to test the shape parameter based on the relationship among some theoretical distributions. Next, we report the results of simulation studies. Finally, the last section presents an analysis of the failure times of the air-conditioning system of Boeing aircrafts to illustrate the proposed testing method.

Relationship between Weibull distribution and other common distributions

Weibull distribution

Let X be a random variable ($r.v.$). The probability distribution function (pdf) of two-parameter Weibull distribution $\mathrm{WE}(\beta,\lambda)$ with the shape and scale parameters $\beta > 0$ and $\lambda > 0$, respectively, is defined as

$$f_X(x) = \lambda\beta x^{\beta-1}\mathrm{e}^{-\lambda x^{\beta}}, \quad x > 0 \tag{1}$$

The Weibull distribution is an exponential distribution $\mathrm{E}(\lambda)$ with scale parameter λ when $\beta = 1$.

If $F_X(x)$ indicates the Weibull distribution function, we will use $\bar{F}_X(x)$ to denote the corresponding survival function. Thus, the survival function $\bar{F}_X(x)$ takes the following form:

$$\bar{F}_X(x) = P(X \geq x) = \mathrm{e}^{-\lambda x^{\beta}}, \quad x > 0$$

and the failure rate function of X, say $r(x)$, is given by

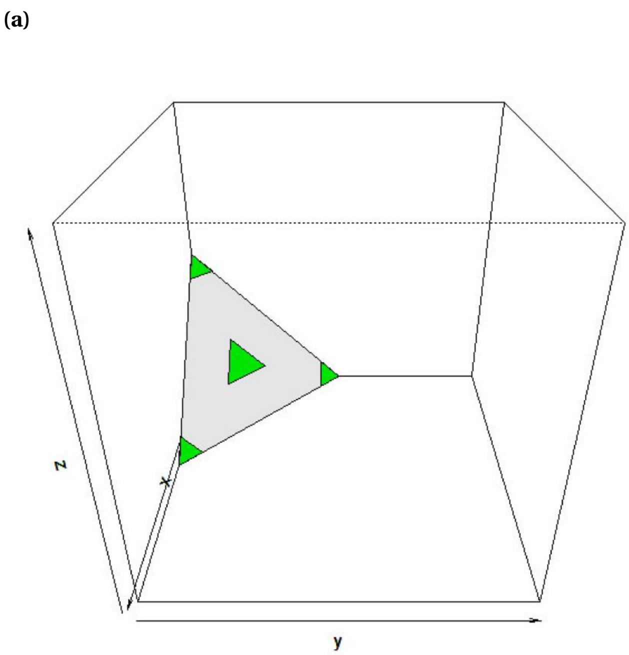
$$r(x) = \lambda\beta x^{\beta-1}, \quad x > 0.$$

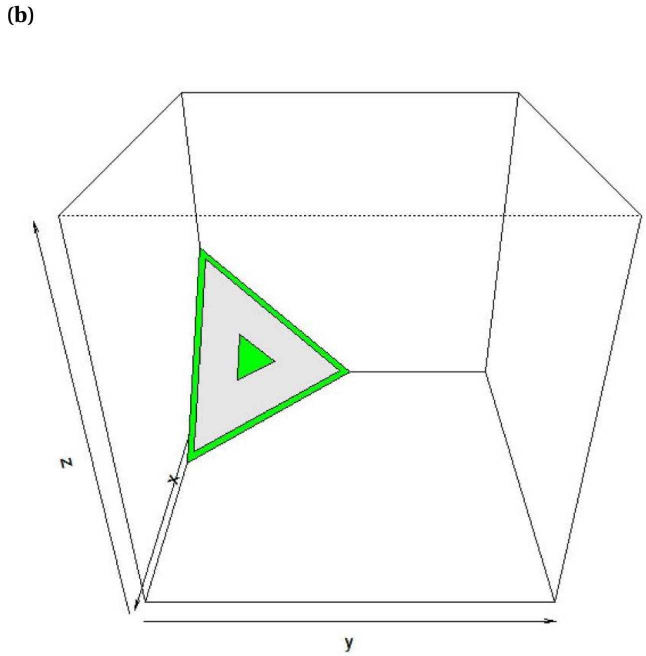
Gamma distribution

Let X be a $r.v.$ The pdf of the two-parameter Gamma distribution $\mathrm{Ga}(\alpha,\lambda)$ with shape and scale parameters $\alpha > 0$ and $\lambda > 0$, respectively, is defined as

$$p_X(x) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)}x^{\alpha-1}\mathrm{e}^{-\lambda x}, \quad x > 0, \tag{2}$$

Figure 1 ■ Vertex (a) and edge (b) methods of the rejection regions of H_0 for $x \in \mathrm{R}^3$





where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} \mathrm{e}^{-x} \mathrm{d}x$ is the gamma function ($\alpha > 0$), such that $\mathrm{Ga}(1,\lambda)$=$\mathrm{E}(\lambda)$.

Dirichlet distribution

The Dirichlet distribution of order $n \geq 2$ with parameters $\alpha_1, \cdots, \alpha_n$ has a pdf with *r.v.* $X = (X_1, \cdots, X_n)$ given by (Kotz, Balakrishnan, & Johnson, 2000)

$$f(x_1, \cdots, x_{n-1}; \alpha_1, \cdots, \alpha_n) = \frac{1}{\mathrm{B}(\alpha)} \prod_{i=1}^{n} x_i^{\alpha_i - 1} \tag{3}$$

which will be denoted by $X \sim \mathrm{D}(\alpha_1, \cdots, \alpha_n)$ on the open $(n-1)$-dimensional simplex defined by:

$$x_1 > 0, \cdots, x_{n-1} > 0$$

$$x_1 + \cdots + x_{n-1} < 1$$

$$x_n = 1 - x_1 - \cdots - x_{n-1}$$

and zero elsewhere.

The normalizing constant $\mathrm{B}(\alpha)$ is the multinomial Beta function, which can be expressed in terms of the gamma function:

$$\mathrm{B}(\alpha) = \frac{\prod_{i=1}^{n} \Gamma(\alpha_i)}{\Gamma\left(\sum_{i=1}^{n} \alpha_i\right)}, \alpha = (\alpha_1, \cdots, \alpha_n).$$

Relations

Theorem 1. Suppose $\{X_i; i = 1, 2, ..., n\}$ is a sequence of independent and identically distributed non-negative *r.v.* Let $X_i \backsim \mathrm{WE}(\beta, \lambda)$. Define $Y_i = X_i^\beta$, then
(a) $Y_i \backsim \mathrm{E}(\lambda)$,
(b) $T = \sum_{i=1}^{n} Y_i \backsim \mathrm{Ga}(n, \lambda)$,
(c) $Z_i = \frac{Y_i}{T} \backsim \mathrm{D}(1, 1, \cdots, 1)$.

Proof:
(a)

$$F_{Y_i}(y) = P(Y_i \leq y) = P(X_i^\beta \leq y) = 1 - \mathrm{e}^{-\lambda y}.$$

(b) (i) When $n = 1$, the proof is immediate.
(ii) When $n = 2$, for $T \leq 0$, $p_T(t) = 0$, whereas for $T > 0$,

$$p_T(t) = \lambda^2 \int_0^t \mathrm{e}^{-\lambda(t-y)} \cdot \mathrm{e}^{-\lambda y} \mathrm{d}y = \lambda^2 t \mathrm{e}^{-\lambda t}, \quad t > 0,$$

such that we have $T \backsim \mathrm{Ga}(2, \lambda)$.

(iii) Furthermore, when $n > 2$, the formula can be extended to $Y_1 + Y_2 + \cdots + Y_n$, we obtain $\sum_{i=1}^{n} Y_i \backsim \mathrm{Ga}(n, \lambda)$ immediately.

(c) The joint pdf of $(T, Z_1, \cdots, Z_n)$ is given by

$$f(t, z_1, \cdots, z_{n-1}) = \begin{vmatrix} \frac{\partial Y_1}{\partial T} & \frac{\partial Y_1}{\partial Z_1} & \cdots & \frac{\partial Y_1}{\partial Z_{n-1}} \\ \frac{\partial Y_2}{\partial T} & \frac{\partial Y_2}{\partial Z_1} & \cdots & \frac{\partial Y_2}{\partial Z_{n-1}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial Y_n}{\partial T} & \frac{\partial Y_n}{\partial Z_1} & \cdots & \frac{\partial Y_n}{\partial Z_{n-1}} \end{vmatrix} \cdot \prod_{i=1}^{n} f_{Y_i}(tz_i) = t^{n-1}\lambda^n e^{-\lambda t}$$

such that we obtain the joint pdf of $(Z_1, \cdots, Z_n)$ as follows:

$$f(z_1, \cdots, z_{n-1}) = \int_0^{\infty} f(t, z_1, \cdots, z_n) \mathrm{d}t = \Gamma(n)$$

Thus, (c) is immediately obtained. ■

Hypotheses test

From Theorem 1, we found that the $r.v.\ Z_i$ only depend on the shape of the Weibull distribution and is independent of the scale parameter λ. Thus, for $r.v. x \sim \text{WE}(\beta, \lambda)$, the following null hypothesis can be considered:

$$H_0 : \beta = \beta_0,$$

where β_0 is the hypothesized shape parameter value.

Suppose

$$\Delta = \{\mathbf{x} = (x_1, \cdots, x_n)' : \sum_{i=1}^{n} x_i = 1, x_i > 0, \mathbf{x} \in R^n\}.$$

Based on (3), the pdf of $\mathbf{X}$ becomes

$$f(\mathbf{x}) = I_{\Delta}(\mathbf{x}) n!, \quad \mathbf{x} \in R^n$$

where $I_{\Delta}(\mathbf{x})$ is the indicator function.

Let

$$\Delta(\gamma) = \{\mathbf{x} = (x_1, \cdots, x_n)' : \sum_{i=1}^{n} x_i = r(\gamma) < 1, x_i > 0, \mathbf{x} \in R^n\}.$$

$$P(\mathbf{X} \in \Delta(\gamma) + a(\gamma)) = \gamma < 1,$$

where $a(\gamma) = \frac{1}{n} - \frac{r^2(\gamma)}{n}, \Delta(\gamma) + a = \{\mathbf{x} : \mathbf{x} = a\mathbf{1} + \mathbf{y}, \mathbf{y} \in \Delta(\gamma)\}$.

Note that the normal direction of the hyperplane $L : \sum_{i=1}^{n} x_i = 1$ is

$$\mathbf{f} = (\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}, \cdots, \frac{1}{\sqrt{n}})',$$

The hyperplane L can also be expressed as

$$\{\mathbf{x} : \mathbf{f}'\mathbf{x} = \frac{1}{\sqrt{n}}\}, \quad \Delta = \{\mathbf{x} : x_i > 0, \mathbf{x} \in L\},$$

The intersection of hyperplane L and normal $\mathbf{f}$ is $(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n})'$, which is called the central point. Δ has n vertices. The i-th coordinate of i-th vertex is described as 1, and the other $n-1$ coordinate of the i-th vertex is 0.

For any point $\mathbf{z}$=$(z_1, \cdots, z_n)' \in \Delta$, if $\beta_0 = 0$, it means z is the central point of Δ, whereas for $\beta_0 = \infty$, z is close to the vertex of Δ or edge of Δ. Therefore, the rejection region of H_0 refers to the areas near to the central point and vertex of Δ or edge of Δ. For example, the rejection regions of the vertex method for $x \in \mathrm{R}^3$ are the green parts shown in Fig. 1(a) and the rejection regions of the edge method for $x \in \mathrm{R}^3$ are the green parts shown in Fig. 1(b).

The key to express the rejection regions is to describe the green parts. For the vertex method, let the area of the central area be $\gamma/2$ and the area of each vertex region be $\gamma/(2n)$. Then, for a given significant level γ, the rejection regions are given by

$$W_v = \{\forall z_i \in \Delta, z_i \geq \frac{1 - \sqrt[n-1]{\frac{\gamma}{2}}}{n}, i = 1, \cdots, n\} \cup \{\exists z_i \in \Delta, z_i \geq 1 - \sqrt[n-1]{\frac{\gamma}{2n}}, i = 1, \cdots, n\}.$$

For the edge method, let the areas of the center and the edge parts be $\gamma/2$.

$$W_e = \{\forall z_i \in \Delta, z_i \geq \frac{1 - \sqrt[n-1]{\frac{\gamma}{2}}}{n}, i = 1, \cdots, n\} \cup \left\{\forall z_i \in \Delta, z_i \geq \frac{1 - \sqrt[n-1]{1 - \frac{\gamma}{2}}}{n}, i = 1, \cdots, n\right\}.$$

Simulation

Hypotheses test on β

We consider the following null hypothesis:

$$H_0 : \beta = \beta_0,$$

under the following conditions of $\lambda = 1, \beta_0 = 0.5$, dimension $n = 10, 30, 40, 50, 100$ and significance level $\gamma = 0.1$. Under these specified values, powers with different dimensions of the hypotheses test are obtained from 3,000 iterations of the Monte Carlo simulation. One simulation procedure is as follows:

1. One random sample $X_i, i = 1, \cdots, n$ of size n is generated from $\text{WE}(\lambda, \beta)$;
2. $Z_i = \frac{X_i^{\beta_0}}{\sum_{i=1}^{n} X_i^{\beta_0}}$ is calculated;
3. if Z_i falls in the rejection regions W_v (or W_e when the second method is evaluated), the null hypothesis is rejected for that sample.
4. Power is calculated as the proportion of rejection out of the 3000 simulation runs.

Table 1 ■ Simulation of significance levels by different methods when $\beta_0 = 0.5$ and 1.5.

		$\beta_0 = 0.5$			$\beta_0 = 1.5$		
n	Method	$\gamma = 0.01$	$\gamma = 0.05$	$\gamma = 0.1$	$\gamma = 0.01$	$\gamma = 0.05$	$\gamma = 0.1$
5	vertex	0.007	0.054	0.101	0.011	0.050	0.103
	edge	0.014	0.054	0.102	0.010	0.052	0.104
	likelihood ratio	0.015	0.057	0.094	0.020	0.057	0.109
7	vertex	0.011	0.045	0.100	0.015	0.047	0.107
	edge	0.009	0.053	0.107	0.010	0.053	0.102
	likelihood ratio	0.012	0.060	0.109	0.016	0.057	0.110
9	vertex	0.009	0.051	0.106	0.009	0.045	0.099
	edge	0.010	0.045	0.103	0.013	0.042	0.107
	likelihood ratio	0.012	0.063	0.108	0.011	0.061	0.108
15	vertex	0.005	0.060	0.100	0.007	0.051	0.099
	edge	0.006	0.055	0.101	0.005	0.047	0.103
	likelihood ratio	0.005	0.063	0.115	0.017	0.062	0.106
30	vertex	0.012	0.042	0.097	0.007	0.052	0.099
	edge	0.008	0.049	0.099	0.010	0.046	0.088
	likelihood ratio	0.013	0.036	0.086	0.014	0.054	0.078

Figs. 2(a) to 2(e) shows the estimated power for the vertex and the edge methods with $n = 10, 30, 40, 50$ and 100 respectively.

From Figs. 2(a) to 2(c), we find that the edge method outperforms the vertex method at either $n = 10$ or 30 when the null hypothesis is $H_0 : \beta = 0.5$. Figs. 2(d) to 2(e) show that the power of the vertex method is higher than the edge method when n is greater than 30.

Finally, Fig. 3 compares power between the likelihood ratio method and the proposed methods using 1,000 iterations of a Monte Carlo simulation under the following conditions of $\lambda = 1, \beta_0 = 0.5, n = 50$, tested for $\beta \in (0.01, 2.5)$ and significance level $\gamma = 0.1$.

Table 2 compares simulated significance levels among the likelihood ratio method and the proposed methods. From the table we can see that the simulated significance levels produced by the vertex method and edge method are closer to the given significance levels than the likelihood ratio method for small sample sizes.

Confidence interval on β

The rejection region and confidence interval are mutually complementary, such that the latter can be derived from the former. Therefore, for a given confidence level $1-\gamma$, a lower bound for the β is the maximum value of it when any z_i satisfies

$$z_i \geq \frac{1 - \sqrt[n-1]{\frac{\gamma}{2}}}{n}.$$

An upper bound for the β is the minimum value of it when exist one z_i satisfies

$$z_i \geq 1 - \sqrt[n-1]{\frac{\gamma}{2n}}.$$

To estimate a lower bound for the shape parameter β, we propose the following steps:

1. Random samples $X_i, i = 1, \cdots, n$ are generate from $\mathrm{WE}(\lambda, \beta_0)$ after 1000 times, where β_0 is the true value of β;
2. β_{L_0} and β_{U_0} are denoted as the initial lower and upper bounds of β, respectively, and
$$\beta = \frac{\beta_{L_0} + \beta_{U_0}}{2}$$
is taken as the initial value of β;
3. $z_i = \dfrac{X_i^\beta}{\sum_{i=1}^{n} X_i^\beta}$ is calculated as well as $length = \beta_{U_0} - \beta_{L_0}$;
4. While $length >$ eps, if for any $i = 1, \cdots, n$, $z_i \geq \dfrac{1 - \sqrt[n-1]{\frac{\gamma}{2}}}{n}$, let $\beta_{L_0} = \beta$ and return to Step 2; else $\beta_{U_0} = \beta$ and return to Step 2;
5. When $length \leq$ eps, $\beta_L = \beta_{U_0}$, and β_L is the lower bound of β.

To estimate an upper bound for the β, we propose the following steps:

1. Random samples $X_i, i = 1, \cdots, n$ are generated from $\mathrm{WE}(\lambda, \beta_0)$ after 1000 iterations, where β_0 is the true value of β;
2. β_{L_0} and β_{U_0} are denoted as the initial lower and upper

Table 2 ■ Summary of the results for the confidence interval of β as a function of the significance level γ for different methods.

n	β_0	Method	$\gamma=0.1$ *length*	$\gamma=0.1$ MSE	$\gamma=0.1$ coverage	$\gamma=0.05$ *length*	$\gamma=0.05$ MSE	$\gamma=0.05$ coverage	$\gamma=0.02$ *length*	$\gamma=0.02$ MSE	$\gamma=0.02$ coverage
30	0.2	vertex	0.2076	0.0177	90.0%	0.2521	0.0236	94.9%	0.2968	0.0308	97.9%
		edge	0.1887	0.0028	89.6%	0.2402	0.0047	95.4%	0.2840	0.0061	98.1%
	0.5	vertex	0.5144	0.1014	90.4%	0.6167	0.1332	95.2%	0.7784	0.2208	98.3%
		edge	0.4831	0.0137	90.1%	0.6177	0.0243	95.8%	0.7169	0.0281	98.3%
50	0.2	vertex	0.1611	0.0068	90.7%	0.1950	0.0091	94.5%	0.2471	0.0139	98.0%
		edge	0.1690	0.0016	89.2%	0.2049	0.0025	94.8%	0.2501	0.0034	98.5%
	0.5	vertex	0.4251	0.0511	89.0%	0.5118	0.0646	95.1%	0.6130	0.0816	97.7%
		edge	0.4247	0.0079	91.8%	0.5166	0.0116	94.9%	0.6424	0.0186	98.2%
75	0.2	vertex	0.1473	0.0058	90.6%	0.1746	0.0062	95.6%	0.2071	0.0081	96.8%
		edge	0.1577	0.0011	89.2%	0.1927	0.0015	95.7%	0.2329	0.0026	97.9%
	0.5	vertex	0.3614	0.0328	90.2%	0.4391	0.0433	95.4%	0.5156	0.0445	97.9%
		edge	0.3947	0.0059	90.3%	0.4850	0.0089	94.9%	0.5836	0.0127	97.8%
100	0.2	vertex	0.1337	0.0040	89.8%	0.1589	0.0050	94.9%	0.1909	0.0056	98.2%
		edge	0.1483	0.0009	90.1%	0.1797	0.0014	95.1%	0.2183	0.0021	98.2%
	0.5	vertex	0.3428	0.0277	91.2%	0.4026	0.0309	94.9%	0.4903	0.0367	98.2%
		edge	0.3764	0.0046	90.8%	0.4546	0.0071	94.7%	0.5596	0.0100	98.1%

bounds of β, respectively, and

$$\beta = \frac{\beta_{L_0} + \beta_{U_0}}{2}$$

is taken as the initial value of β;

3. $z_i = \dfrac{X_i^\beta}{\sum_{i=1}^n X_i^\beta}$ is calculated as well as $length = \beta_{U_0} - \beta_{L_0}$;
4. While $length >$ eps, for the vertex method, if exist $i = i_0$, $z_{i_0} \geq 1 - \sqrt[n-1]{\dfrac{\gamma}{2n}}$, let $\beta_{U_0} = \beta$ and return to Step 2; else $\beta_{L_0} = \beta$ and return to Step 2. For the edge method, if for not all $i = 1, \cdots, n$, $z_i \geq \dfrac{1 - \sqrt[n-1]{1-\frac{\gamma}{2}}}{n}$, let $\beta_{U_0} = \beta$ and return to Step 2; else $\beta_{L_0} = \beta$ and return to Step 2.
5. When $length \leq$ eps, $\beta_U = \beta_{L_0}$, and β_U is the upper bound of β.

Thus, for a given confidence level $1-\gamma$, the confidence interval for β is

$$[\beta_L, \beta_U] \tag{4}$$

To test the performances of these estimation methods, Monte Carlo simulations were executed using several samples of size n, and different shape parameters. Without loss of generality, the chosen values for n were 30, 50, 75 and 100. In addition, the scale and shape parameters were fixed at $\lambda = 1$, $\beta_0 = 0.2$ and 0.5. For different combinations of sample size, n, and β_0, the Weibull distribution samples were generated by performing 1000 simulations in each case. To verify the accuracy of the proposed methods, we calculated the actual coverage of the confidence interval with the formula: coverage $= N_0/N$ where N_0 represents the number of β falling into the confidence interval. The results of the simulations are summarized in Table 1.

The results in Table 1 indicate that:

1. Among the different methods, the edge method outperforms the vertex method under almost all sample sizes and β_0.
2. At different significance levels, the actual coverage rate approximates well the corresponding confidence level. The proposed methods for estimating confidence intervals is thus efficient. In addition, the result is satisfactory for decreasing γ.
3. As expected, with increasing sample sizes, the mean length of the confidence interval and its MSE decrease.

Table 3 ■ Summary of the results for the null hypothesis: $H_0 : \beta = \beta_0$ by the vertex and the edge methods.

β_0	γ	H_0 vertex	H_0 edge
0.5	0.01	reject	reject
2	0.01	accept	accept
4	0.01	accept	reject
0.5	0.05	reject	reject
2	0.05	accept	accept
4	0.05	reject	reject
0.5	0.1	reject	reject
2	0.1	accept	accept
4	0.1	reject	reject

One application

Proschan (1963) reported the failure times of the air-conditioning system of 13 Boeing aircrafts. Based on the report, we use the data set associated with Plane 7909 in Proschan (1963). The successive failure times are: 90, 100, 160, 346, 407, 456, 470, 494, 550, 570, 649, 733, 777, 836, 865, 983, 1008, 1164, 1474, 1550, 1576, 1620, 1643, 1705, 1835, 2043, 2113, 2214, and 2422 h.

From (4) for the vertex method, a 90% confidence interval for β is [0.932, 3.541], whereas a 95% confidence interval for β is [0.858, 3.940]. We can consider the following hypothesis:

$$H_0 : \beta = \beta_0$$

for the following conditions of $\beta_0 = 0.5, 2, 4$ and significance level $\gamma = 0.01, 0.05, 0.1$. The results of the hypotheses are shown in Table 3 using the two different testing methods.

Conclusion

In this article, we proposed two new methods for testing hypotheses on the shape parameter of a two-parameter Weibull distribution. The test statistics are constructed based on the relationships among Gamma distribution, Dirichlet distribution, and Weibull distribution. We obtain two rejection regions and confidence intervals in $n-$dimensional space by analyzing the variation trends of the shape parameter and by using properties of the Dirichlet distribution. The performances of the testing methods were investigated using Monte Carlo simulations. The methods are observed to be satisfactory. The coverage of the confidence interval is close to the confidence level, and the two proposed methods exhibited satisfactory performance.

Authors' note

This work is supported by the Research Fund for the Doctoral Program of Higher Education of China under Grant 20131103120027, the Beijing Natural Science Foundation Grant No.1122001, the National Natural Science Foundation of China through Grant No. 11072007, No. 11372014, the International Science and Technology Cooperation Program of China through Grant No. 2014DFR61080.

References

Bai, X. P. & Mu, J. J. (2011). Research on three-parameter weibull distribution in failure data processing and dynamic reliability evaluating of belt transportation system in mines [in chinese]. *J. Nanjing University of Science and Technology*, *35*, 97–101.

Bain, L. J. & Engelhardt, M. (1980). Inferences on the parameters and current system reliability for a time truncated weibull process. *Technometrics*, *22*, 421–426. doi:10.1080/00401706.1980.10486174

Duffy, S. F., Starlinger, A., & Powers, L. M. (1993). Reliability analysis of structural ceramic components using a three-parameter weibull distribution. *Journal of Engineering for Gas Turbines and Power*, *11*, 109–116. doi:10.1115/1.2906664

Kotz, S., Balakrishnan, N., & Johnson, N. L. (2000). *Continuous multivariate distributions, volume 1: models and applications*. New York: Wiley.

Kuo, L. & Yang, T. Y. (1996). Bayesian computation for nonhomogeneous poisson process in software reliability. *Journal of the American Statistical Association*, *91*, 763–773. doi:10.1080/01621459.1996.10476944

Lockhart, R. A. & Stephens, M. A. (1994). Estimation and tests of fit for the three-parameter weibull distribution. *Journal of the Royal Statistical Society, Series B*, *56*, 491–500.

Proschan, F. (1963). Theoretical explanation of observed decreasing failure rate. *Technometrics*, *5*, 375–383. doi:10.1080/00401706.1963.10490105

Ren, G. C., Yang, Z. W., & Meng, B. M. (2012). Reliability evaluation on machining center based on three-parameter weibull distribution. *Advanced Materials Research*, *430*, 1645–1649. doi:10.4028/www.scientific.net/AMR.430-432.1645

Ryan, K. J. (2003). Some flexible families of intensities for nonhomogeneous poisson process models and their Bayes inference. *Quality and Reliability Engineering International*, *19*, 171–181. doi:10.1002/qre.520

Sürücü, B. & Sazak, H. S. (2009). Monitoring reliability for a three-parameter weibull distribution. *Reliability Engineering and System Safety*, *94*, 503–508. doi:10.1016/j.ress.2008.06.001

Verma, A. K. & Kapur, P. K. (2006). *Quality, reliability and information technology: trends and future directions*. New Delhi: Narosa Publishing House.

Citation

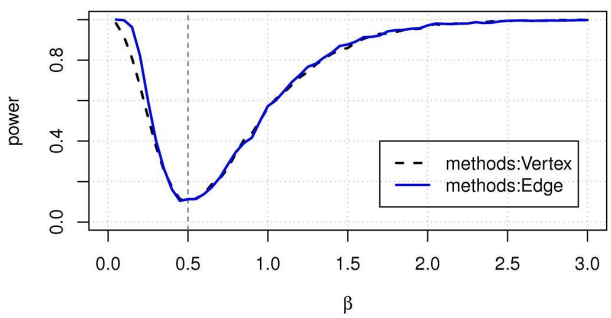
Zhao, X., Cheng, W., Zhang, Y ., Zhang, Q., & Yang, Z. (2015) New statistical inference for the Weibull distribution. *The Quantitative Methods for Psychology*, *11*(*3*), 139-147.

Received: 06/08/2015 ~ Accepted: 26/08/2015

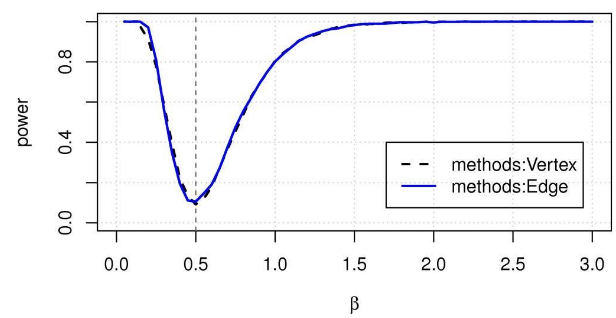
Figures follows on next page

Figure 2 ■ Statistical power of vertex and edge methods for various sample sizes (panels) using significance level $\gamma = 0.1$. The vertical dash line is the position of the true shape parameter.

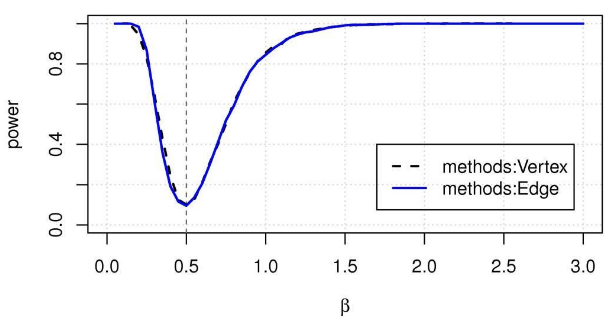
(a) $n = 10$



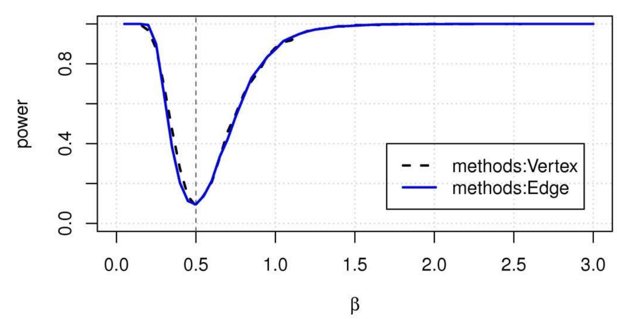
(b) $n = 30$



(c) $n = 40$



(d) $n = 50$



(e) $n = 100$

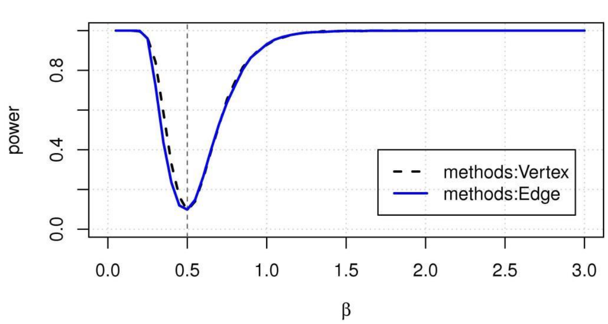


Figure 3 ■ Power of $H_0 : \beta = \beta_0$ for true $\beta = 0.5$, and tested at $\beta_0 \in (0.01, 2.5)$