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Dynamic structures of parent-child number talk: An application of categorical cross-recurrence quantification analysis and companion to Duong et al. (2024)

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Abstract ■ Social interactions, particularly parent-child conversations, play a critical role in children's early learning and pre-academic skill development. While these interactions are bidirectional, complex, and dynamic, much of the research in this area tends to separate speakers' talk and capture the frequency of words or utterances. Beyond the aggregation of talk exists rich information about conversational structures and processes, such as the extent to which speakers are aligned or reciprocate each other's talk. These measures can be derived using categorical cross-recurrence quantification analysis (CRQA), a method that quantifies the temporal structure and co-visitation of individual and sequential events, e.g., utterances between speakers. In this paper, we present an application of CRQA, following the protocol described in our tutorial paper (Duong et al., 2024, this issue), to describe alignment in parent-child conversations about numbers and math (i.e., number talk). We used the 'crqa' package in R and the code used in this application is available in the Supplemental Materials. Further, the CRQA measures derived from this application were compared to traditional frequency measures of talk, i.e., counts of utterances, in the prediction of children's math skills. Overall, we showed that (1) CRQA can be applied to existing transcription data to uncover theoretically-driven patterns of parent-child talk that are not captured by common frequency measures and (2) these CRQA measures offer additional, rich information about interactions beyond frequencies of talk and can be used to predict individual differences in children's math skills.

Keywords ■ parent-child interactions, number talk, math skills, recurrence quantification analysis.
Tools ■ R.

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Introduction

Social interactions play a critical role in children's early learning and pre-academic skill development in various domains, including mathematics (Casey et al., 2018; Fox et al., 2023; Gibson et al., 2020; Mutaf-Yıldız et al., 2020; Reynolds et al., 2019), science (Alexandre et al., 2022; Bustamante et al., 2020), and language and literacy (Levickis et al., 2022; Reynolds et al., 2019; Sénéchal & LeFevre, 2002). For instance, in the domain of mathematics, the quantity and quality of parent-child conversations about numbers and math concepts (i.e., number talk) across a range of activities, such as meal times, joint book viewing, and pretend grocery shopping, have been linked with children's concurrent and growth in math skills (e.g., Casey et al., 2018; ; see Silver et al., 2024, for a meta-analysis). Importantly, children's emerging academic skills relate to their later success in school, as well as their health, occupational, and financial outcomes in adulthood (Hanushek & Woessmann, 2015). Thus, understanding the mechanisms underlying the link between parent-child conversations and children's (pre-)academic skills is critical to supporting child development.

Studies of dyadic interactions in the context of child development have been dominated by methods that condense or collapse rich observational data into aggregate measures of talk, such as the quantity of words or utterances from an individual (e.g., Reynolds et al., 2019). However, some of our assumptions and knowledge about how parent-child interactions contribute to early learning are based on the idea that interacting partners exhibit mutual alignment (hereafter referred to as alignment), a process involving the reciprocal exchange of information and the adaptation of behaviors in varying degrees and patterns. Thus, social interactions may be better characterized at the level of the dyad or group rather than the individual (Gallotti et al., 2017), and traditional frequency measures like counts of words or utterances may not adequately capture what researchers assume is occurring within interactions.

One avenue for expanding on these common frequency data to capture conversational structures and processes, i.e., *how* parent and child talk *interact*, is to examine patterns of language input at other levels of granularity. Specifically, beneath their aggregation, we may seek to understand the dynamics of conversations, such as speakers' alignment, the consistency of talk across time, and how introducing certain types of talk affects subsequent talk. Extending existing measures of parent-child interactions acknowledges their dynamics and richness and has the potential to uncover or clarify the processes that drive the relations between parent-child conversations and children's early academic skills. In this paper, following the procedure outlined in our tutorial paper (Duong, Davis, et al., 2024), we demonstrate an application of cross-recurrence quantification analysis (CRQA) and the utility of capturing alignment in dyadic interactions. First, we examine qualitative differences between descriptions of parent-child interactions that involve conversations about numbers and math (i.e., number talk) using CRQA and traditional frequency measures, i.e., counts of number talk utterances. Then, we investigate whether CRQA measures provide more explanatory power than frequency variables in predicting child outcomes.

Theoretical underpinnings of social interactions about numbers and math

Early math learning through parent-child interactions is bidirectional, complex, and dynamic, and several interrelated processes may underlie the relations between number talk and children's developing math skills. For instance, children's exposure to others' and their own math language may support their acquisition of math concepts through repeated and focused practice of various number skills, such as identifying Arabic numerals, counting, labeling sets of items, comparing magnitudes, and performing arithmetic functions (e.g., Lehrl et al., 2020; Levine et al., 2010; Mutaf-Yıldız et al., 2020; Ramani et al., 2015). Consequently, children's attention to number or interest in math may grow (along with their skills), which can positively impact their capacity to respond to and initiate number talk (Batchelor et al., 2015).

These propensities to engage in talk about numbers, or informal math conversations and learning more broadly, may be brought on by micro-level events that occur during dyadic conversations. This view is consistent with a dynamic systems perspective on social interactions and learning, which proposes that conversations between two individuals or systems are adaptive, complex, and sensitive to feedback (e.g., Solomon et al., 2021). One speaker's subsequent linguistic input is influenced by other's previous patterns of input and characteristics of the other person, such as their cognitive abilities (e.g., Cox & van Dijk, 2013; Denby & Yurovsky, 2019; Fusaroli et al., 2021; Misiek et al., 2020; Yurovsky et al., 2016). For instance, parents and children may adjust their number talk in real-time and employ various types of talk throughout (e.g., questions and statements focused on different math concepts) based on the other person's contributions. Also, parents' subsequent number talk may be driven by their child's responses, which can potentially serve as signals of their engagement or capacity to respond to math-related input. Parents' repeated number input may indicate to their child that math-related topics are important or relevant to discuss during everyday activities, which may impact their child's later number talk. This reciprocity and alignment between parents and children may reinforce children's existing number knowledge and scaffold their acquisition and practice of number skills. Some of these processes were highlighted in recent work by Eason and colleagues (2021), who found that parent number prompts such as questions tended to result in lengthier discussions of number-related topics with their children than statements containing number words. Broadly, their findings suggest that interactions between children and their parents have a structure to them, which can be observed in how one speaker's linguistic input influences the other's responses and how both speakers subsequently interact with each other.

Cross recurrence quantification analysis (CRQA) of number talk

To capture alignment in parent-child conversations, researchers can apply cross-recurrence quantification analysis (CRQA). CRQA is a non-linear data analysis method that visualizes and quantifies the frequency and duration of repeated events (called recurrences) in dynamical systems—such as interacting speakers. Specifically, CRQA measures both the degree and temporal structure of moments of co-visitation between two interacting systems, where *co-visitation* refers to one system entering a state or event previously experienced by another. Given our interest in number talk, we can define our state of interest as a "number talk utterance". Then, a co-visitation occurs, for example, if a parent speaks a number related phrase previously mentioned by a child, whether immediately in the next sentence or after some delay.

These moments of number talk co-visitations, or *cross-recurrences*, may be visually mapped using a cross-recurrence plot (CRP)—a two-dimensional representation of the behaviors of the interacting persons across time. The CRP displays cross-recurrences in the form of a binary matrix where each axis represents the timeline of utterances of one individual. In this matrix, values of 1 represent moments of cross-recurrence, signifying that *both* parent and child were engaged in number talk at some lag of time. In contrast, values of 0 represent moments of non-recurrence where only one or neither person was engaged in number talk. Typically, areas in the CRP corresponding to each value are represented with different colors (e.g., recurrences or 1 in purple and non-recurrences or 0 in grey; see Figure 1 from Duong et al., 2024, for examples of CRPs). The distance between recurrent points in this matrix indicates the amount of time between co-visitations, where adjacent recurrent points indicate successive moments of number talk (e.g., a parent utters number talk immediately after the child) and greater spaced points indicate longer periods between reciprocal number talk engagement (e.g., a parent utters number talk several sentences later). The quantification of a CRP produces several descriptive measures of the interaction which are described below.

Table 1 from our tutorial paper (Duong, Davis, et al., 2024) provides a summary of these CRQA metrics and their interpretation. The recurrence rate (RR) can be derived to represent the alignment or reciprocation of number talk. In our case, the RR would refer to the relative frequency with which both speakers used number talk over time out of all possible opportunities for this talk to recur. This differs from examining the proportion of number talk utterances out of total utterances in an interaction because the number of possible opportunities in CRQA is the area of the CRP. It is possible for a dyad to have high occurrences of number talk and low recurrence, particularly if the number talk utterances were primarily spoken by one person and their partner was not reciprocating. Further, in this context, determinism (DET) and mean diagonal line (MeanL) describe the extent to which dyads engage in "back and forth" exchanges, i.e., conversational turns, of number talk and the length of these exchange sequences, respectively. For example, a diagonal line of length = 3 would be formed in the following three utterance exchange, where each contains number talk:

- Parent: "Would you like one apple?"
- Child: "No, I want two."
- Parent: "Here are your two apples."

In contrast, a lack of diagonal lines can occur when there are periods between reciprocal engagement. For example, no diagonal line would be formed in this exchange, as the child breaks the back-and-forth sequence:

- Parent: "Would you like one apple?"
- Child: "I love apples!"
- Parent: "Then would you like two apples?"

Laminarity (LAM) and trapping time (TT) reflect the relative occurrence and length of consecutive number utterances by either speaker, respectively. In other words, one speaker initiates number talk and the other engages in successive instances of number talk:

- Parent: "Would you like one apple?"
- Child: "No, I want two."
- Child: "Can I also have one pizza?"

Thus, if LAM is high, this reflects a greater tendency to engage in consecutive bouts of number talk. Overall, these measures provide a sense of whether and how parent and child number talk is concentrated in clusters of sequential utterances.

More generally, all of these CRQA measures describe how both speakers interact, going beyond common frequency measures that essentially separate parent and child number talk from each other. Thus, examining dyads' CRPs and variability in the derived CRQA measures may provide additional insight into the mechanisms underlying the link between number talk and children's early math abilities.

Related literature

The present work is informed by a growing body of research in children's cognitive development which has applied CRQA to studying how features of adult-child conversations relate to children's early language acquisition and learning. Indeed, social interactions are crucial for language development (e.g., Lytle & Kuhl, 2018) and these interactions contain meaningful linguistic structures and patterns (e.g., Grosz & Sidner, 1986; Kyratzis, 2017; Rohde & Frank, 2011). Much of this work has made use of widely available corpus data of parent-child interactions in naturalistic settings (e.g., from the CHILDES database; MacWhinney, 1992) and typically tracked changes in conversational patterns, i.e., CRQA measures, as a function of time or development (Abney et al., 2017; Cox & van Dijk, 2013; Dale & Spivey, 2005, 2006; de Wolf, 2014; Fernandez & Grimm, 2014; Grimm & Fernandez, 2014; Leonardi et al., 2016). More recent work has examined how CRQA-derived features of interactions between infants and parents differ based on cultural background and the number of languages spoken in the home (Gampe et al., 2020), and how CRQA variables describing teacher-student interactions change as a function of an educational intervention (Menninga et al., 2019). There is also a large body of work investigating other communicative behaviors between dyads such as gazes, head and hand movements, and other joint actions (e.g., López Pérez et al., 2017; Rohlfing et al., 2020; Xu & Yu, 2016). Here, we focus on reviewing studies that analyze language input from learning partners, specifically parents (and teachers) and young children, during everyday activities in the home.

The earliest applications of CRQA to studying parent-child conversations used CHILDES corpus data (MacWhinney, 1992) and examined linguistic alignment (sometimes referred to as "convergence" or "coordination") at the level of *words*. Specifically, Dale and Spivey (2005, 2006) explored three dyads' lexical and syntactic alignment during multiple interactions in the home and whether recurrence varied by temporal proximity and child age. The three corpora were Brown's Sarah from ages 2 years 9 months to 5 years 1 month old (Brown, 1973), Kuczaj's Abe from 2 years 5 months to 5 years 0 months (Kuczaj, 1976), and Sachs' Naomi from 1 year 11 months to 4 years 8 months (Sachs, 1983). Lexical alignment referred to the recurrence of individual words or tokens (e.g., "the" or "you") used by parents and children, and syntactic alignment referred to the re-visitation of word classes or parts of speech (e.g., verb, noun, pronoun, determiner). The researchers compared dyads' overall word-level alignment (global RR) with their local alignment within a specific temporal window (specifically RR at 50 words above and below the line of incidence). In sum, lexical and syntactic recurrence about the LOI was greater than global RR, suggesting that alignment tended to be stronger between words that were closer to each other in time. Moreover, global RR and recurrence along the LOI decreased with child age, suggesting that in the early stages of development, parents may be guiding and adapting to children's use of words and word classes, but as children age, this adaptation diminishes because children's array or knowledge of lexical and syntactic items expands (Dale & Spivey, 2005).

Later, using the same sample, Dale and Spivey (2006) investigated age-related changes in the temporal ordering or "leading" of syntactic alignment by quantifying recurrence above and below the LOI. These patterns of leading were visualized in the upper and lower triangles of the CRP; with the parent's data series on the x-axis and the child's data on the y-axis, the upper triangle represented the child's word usage following parent usage, and the lower triangle reflected the parent's input following child input. A greater RR in the upper triangle suggested that the parent tended to lead the conversation, and a greater RR in the lower triangle vice versa. The researchers found individual differences in the extent to which parents and children led their conversations across development. For instance, Abe's tendency to lead conversations increased with age, whereas Sarah consistently followed her parent in their conversations (Dale & Spivey, 2006). Overall, Dale and Spivey (2005, 2006) demonstrated that parent-child conversations in everyday, naturalistic settings are coordinated at the word-level, that this alignment tends to diminish with development, and that differences in patterns of leading may correspond to variability in the developmental level of the child.

Later work by Cox and van Dijk (2013) also utilized CHILDES corpus data and measured recurrence at the utterance level. Specifically, they analyzed changes in the dynamics of naturalistic observations ($n = 22$) of one parent-child dyad as a function of child age from 1 year 7 months to 2 years 6 months old. Each utterance was assigned one of three categories: (a) one-, (b) two- or three-, or (c) four- or more word utterance, and the recurrence of utterance lengths was examined. In addition to capturing global RR, the researchers derived measures of consecutive recurrent input, such as laminarity (LAM), trapping time (TT), and maximal vertical length, as well as measures of linguistic exchanges, including determinism (DET), mean diagonal length (meanL), and maximal diagonal length. While the RR, DET, and maximal diagonal length remained relatively stable across time, all other recurrence measures decreased with age. The researchers posited that, with development, the lengths of speakers' utterances may become less predictive of each other because parents' scaffolding tendencies, which involve adapting their input to the child's, naturally decrease as children's ability to verbally convey their own thoughts and ideas improves (Cox & van Dijk, 2013).

Similar work by Menninga et al. (2019) examined changes in the syntactic coordination of teachers and kindergarten students over the course of a three- to four-month long science education intervention. They coded the sentence complexity of student and teacher utterances based on the number of clauses in each sentence: utterances either contained no clause, one clause (simple sentence), or multiple clauses (complex sentence). Over the course of the intervention, teacher-student syntactic alignment of simple sentences decreased while alignment of complex sentences increased (Menninga et al., 2019). These results generally aligned with previous work showing that a decrease in dyadic alignment may correspond to developmental maturation in domain-general and domain-specific (e.g., science) language abilities.

Overall, the aforementioned body of research has shown that adult and child alignment at various levels of linguistic input changes (i.e., generally decreases) with age and cognitive development and that the extent of this alignment, as well as the strength and direction of the aforementioned associations, varies considerably between dyads. These studies showcase the utility of using CRQA to uncover important signatures of conversational structures and processes, such as repetition or imitation, acknowledgements, feedback, and adaptation (e.g., Dale & Spivey, 2005) that may be shaped by children's language abilities and/or adults' general tendencies when interacting with children. The recurrence-related mechanisms of children's early language learning through conversations with their parents may parallel the processes that underlie the contribution of other types of talk, i.e., number talk, to children's learning and development.

The present study

Given our hypothesized underpinnings of the relation been parent-child number talk and children's math skills, and past research using CRQA to study parent-child conversations which revealed developmental changes in the topology of these interactions that corresponded to children's maturation in domain-general and domain-specific (e.g., science) language abilities (e.g., Menninga et al., 2019), we have two research goals:

First, we demonstrate an application of CRQA to capture alignment in parent-child conversations involving number talk (following our tutorial, Duong et al., 2024). Specifically, we derive CRQA measures of parent-child alignment of number talk utterances and illustrate, through qualitative analyses of six example dyads, how these measures provide more information about dyadic interactions over counts of utterances.

Second, we explore whether the link between parent-child number talk and children's math skills is better explained by the derived CRQA measures than traditional count measures.

Method

Participants

Parents and their 4-year-old children ($n = 127$) enrolled in the Parents Promoting Early Learning (PPEL) study, which examines socioeconomic variability in the home learning environment and children's early academic skills (for papers published with these data, see Bachman et al., 2020, 2022; Duong, Davis, et al., 2024; Duong et al., 2021; Elliott et al., 2023; Fox et al., 2023). The study protocol was approved by the local institutional review board and parents gave written informed consent to participate in the study prior to completing any research activities. Three parent-child dyads were excluded from the analyses due to the presence of an additional parent during the observational task ($n = 1$) or missing observational data ($n = 2$), resulting in a final sample of 124 dyads (Mean child age = 4.40 years, SD = .30 years). Families were recruited from a large mid-Atlantic metropolitan area of the United States through community flyer distributions, institutional research participant databases, and recruitment efforts by the research team at preschools and childcare centers. Parents were primarily mothers (85%) and White and non-Hispanic (79%), followed by Black (11%), Asian or Pacific Islander (4%), Hispanic/Latino (2%), or another race or ethnicity (4%). Additionally, parents reported completing 16.11 years of education on average (SD=2.15 years, Range = 11-18 years), with 19% having completed less than a bachelor's degree, 18% having a bachelor's degree, and 56% having more than a bachelor's degree (the rest did not report this information). Families' mean yearly household income was $106,169.70 (SD=$69,114.48, Median=$98,5000, Range=$5,000-$350,000).

Materials and procedure

Semi-structured observation and number talk coding. Parents and children engaged in a pretend grocery shopping activity with a set of developmentally appropriate toys, including pretend food items, fake bills and coins, and a cash register (Figure 1), and they were instructed to play with the materials as they normally would for about 8 minutes. Most of these interactions occurred in dyads' homes, but 2 dyads participated in this semi-structured activity in our lab. Each parent-child interaction was video recorded and transcribed using Datavyu (Datavyu Team, 2014) at the utterance level (Pan et al., 2004) by trained coders.

Coders reviewed each transcript for instances of number talk based on a previously used coding scheme (Bachman et al., 2020), which included discussions of arithmetic, patterns, comparing magnitudes, ordinal relations, counting, identifying number symbols, labeling sets of objects, and other number or math concepts (e.g., time). They searched and evaluated key terms, including number words (e.g., "four"), ordinal words (e.g., "third"), elicitations (e.g., "how many"), and terms associated with specific math concepts (e.g., "count," "add," "take away"). Specifically, a script written in the Ruby programming language automatically extracted utterances containing these terms, and then coders determined whether these utterances were instances of number talk. Utterances containing the search terms but were not used in a math context were not coded as number talk. For instance, utterances that used "one" to refer to an object rather than the quantity (e.g., "You want that one?") or "count" to refer to something other than enu-

Figure 1 ■ Pretend grocery toys used during the parent-child play interactions

meration (e.g., "That time didn't count, I want to go again") were not considered number talk. Number utterances containing more than one relevant search term were coded as an individual instance of number talk and thus, the unit of analysis in this coding scheme was at the utterance (vs. the word) level. See the Supplementary Materials for the number talk coding manual. Coders were highly reliable at identifying instances of parent and child number utterances (κ=.94 for both parent and child talk; percent agreement=97%).

A handful of traditional frequency measures were derived from these parent-interactions, including the total count of utterances spoken by the parent and child (separately) and the total count of number talk utterances by each speaker. Then, following the protocol outlined in our tutorial paper (Duong, Davis, et al., 2024), we used the 'crqa' package in R (Coco et al., 2021) to apply CRQA to these transcription data. We derived measures of parent-child alignment, namely the recurrence rate (RR), determinism (DET), mean diagonal line (meanL), laminarity (LAM), and trapping time (TT). The analysis code for this paper can be found at github.com/s-duong/crqa-number-talk.

Child outcomes

Math Skills. Children completed the Woodcock-Johnson Applied Problems subtest ($r = .93$) (Woodcock et al., 2001), an assessment of their ability to solve practical math problems. The problems were displayed in a binder with accompanying images or text, and they became progressively more challenging with earlier items requiring basic number skills such as counting and later items requiring arithmetic and knowledge of shapes, currency, time, and other units of measurement. The dependent measure was an age-standardized score with an expected mean of 100 and standard deviation of 15, which was z-scored for analyses.

Vocabulary. Parents completed the Developmental Vocabulary Assessment for Parents (DVAP; Libertus et al., 2015), where they were presented with a list of 212 nouns, verbs, and adjectives in variable difficulty and instructed to mark which of the listed words they had heard their child utter. The DVAP is based on Form A of the Peabody Picture Vocabulary Test (PPVT-4 Dunn & Dunn, 2012) and is a valid indicator of children's expressive vocabulary skills (Libertus et al., 2015). Parents completed this report on a paper copy or electronically via the Qualtrics platform, and they were asked to refrain from confirming with their child whether they knew the listed words. The dependent measure was the number of words parents reported as having heard their child say and this measure was z-scored for analyses.

Executive Function. The dimensional change card sort (DCCS) task (Zelazo, 2006) was used to measure children's

Table 1 ■ Summary statistics of overall number talk and child outcomes (Non-imputed data)

Measure	n	Mean	SD	Median	Range	Skew	Kurtosis
Total parent utterances	124	155.14	49.53	150.00	7-408	1.59	6.70
Total child utterances	124	101.99	36.23	100.50	12-241	0.43	0.70
Parent number utterances	124	21.20	14.43	18.00	0-67	1.09	0.94
Child number utterances	124	13.55	11.59	10.00	0-78	2.18	7.21
Child math (WJ AP score)	118	115.14	12.41	116.50	70-147	-0.61	1.79
Child vocabulary (DVAP score)	111	105.58	29.91	106.00	11-179	-0.05	0.37
Child executive function (DCCS score)	119	14.71	5.71	18.00	5-24	-0.62	-1.17
Child age (years)	124	4.40	0.30	4.32	4-4.98	0.34	-1.23

executive function. Participants were shown cards containing images of a boat or rabbit, in blue or red, as well as two boxes; one box was labeled with a picture of a blue rabbit and the other with a red boat. Children completed three phases in which they were instructed to sort the cards according to a particular dimension (i.e., color first, then shape). In the third phase, they were shown cards with or without a border and instructed to sort by either the color or shape dimension ("If there is a border, we play the color game. If there is no border, we play the shape game."). Children were required to sort at least 4 out of 6 cards correctly before moving onto the next phase and they completed up to 24 trials: 6 in each color/shape phase and 12 in the border phase for which they received one point per correct card sort. If children did not advance to a phase, they were assumed to receive a score of 0 for that phase. For instance, a child who only sorted two cards correctly in the first phase received a score of 2 for the entire task. The dependent measure was the percentage of correct trials out of 24, which was z-scored for analyses.

Age. Parents provided their child's birth date as part of an electronic questionnaire through the Qualtrics platform and child age in months was calculated from the first assessment.

Statistical Plan

Patterns of missing child assessment data were examined, revealing that DVAP scores had the greatest number of missing cases ($n = 13$ or 10.48% of the observations), followed by WJ AP scores ($n = 6$ or 4.84%) and DCCS scores ($n = 5$ or 4.03%). Missing child assessment data were imputed using multiple imputation by chained equations (MICE) with the 'mice' package in R (Buuren & Groothuis-Oudshoorn, 2011). Specifically, 40 datasets were created in which missing data were estimated using random sampling from observed values. Descriptive statistics of parent and child talk variables and the child outcomes are shown below in Table 1.

To address the first research aim, we applied CRQA to transcription data of the parent-child interactions to derive their CRPs and several recurrence metrics, including the RR, DET, mean diagonal line, LAM, and TT. Then, we identified several qualitatively different parent-child interactions to explore further, specifically dyads whom had similar frequencies of number talk but differed across the recurrence metrics. These dyadic interactions were further described after linking the content of their conversations to corresponding sections of their CRP (e.g., examining what one parent-child dyad discussed when they showed high DET).

To address the second research aim, pooled zero-order correlations between the number talk frequency and CRQA variables were examined. Then, three sets of pooled stepwise regression models were estimated to predict children's math, vocabulary, and EF skills separately. In the first step of these models, total (parent and child) number talk and child age were entered as predictors of each of children's skills. In the second step, RR, DET, and LAM were added as predictors. Model estimates were compared to investigate whether number talk, specifically the alignment in parent-child number talk, is a potential domain-specific social process of children's early math development.

Results

Research aim #1: Cross-recurrence quantification analysis of number talk

The alignment of parent-child number talk utterances in our sample showed considerable variability. The mean RR was slightly above 1 percent and ranged from 0 to 7.33 percent, suggesting that for many dyads, their interactions mostly consisted of non-number utterances. The mean DET was 16.17 percent and ranged from 0 to 54.71 percent, while the mean LAM was relatively higher at 21.52 percent and ranged from 0 to 63.69 percent. Thus, on average, dyadic conversations consisted of slightly more consecutive number utterances from one speaker after the other speaker used number talk than "back and forth" number exchanges between speakers. Table 2 shows additional descriptive statistics of these CRQA-derived parent-child num-

Table 2 ■ Summary statistics of CRQA-derived parameters of parent-child number talk ($n = 124$)

Measure	Mean	SD	Median	Range	Skew	Kurtosis
Recurrence rate (%)	1.09	1.32	0.67	0.00 – 7.33	2.17	5.49
Determinism (%)	16.17	12.46	15.97	0.00 – 54.71	0.47	−0.35
Mean diagonal line	1.77	0.81	2.00	0.00 – 3.00	−1.59	0.86
Laminarity (%)	21.52	16.34	22.69	0.00 – 63.69	0.24	−0.80
Trapping time	1.91	1.18	2.11	0.00 – 5.39	−0.33	−0.07

Table 3 ■ CRQA parameter values and frequencies of number talk associated with six example dyads

	Example dyad number					
Measure	1	2	3	4	5	6
Total parent utterances	134	112	160	144	164	89
Total child utterances	56	112	106	108	115	75
Parent number utterances	50	6	23	21	25	17
Child number utterances	27	0	6	16	17	10
Recurrence rate (%)	7.33	0	0.28	1.06	1.09	1.26
Determinism (%)	33.18	0	1.45	35.71	5.18	54.71
Mean diagonal line	2.39	0	2	3	2	2.11
Laminarity (%)	34.47	0	26.09	63.69	17.88	23.53
Trapping time	2.44	0	3	4.76	2	2

ber talk variables.

The recurrence plots (CRPs) of six qualitatively different interactions, along with their associated CRQA parameter values and frequencies of number talk utterances (Table 3), are discussed below. The CRP visualizes recurrences of parent and child number talk (in purple) with the parent event series on the x-axis and the child event series on the y-axis. Each CRP includes tick marks on the x- and y-axes indicating occurrences of parent (in blue) and child (in red) number talk as a function of time, i.e., the observation number or the order in which the utterance appeared in the transcript. The line of incidence (LOI; black diagonal line on the CRP) is also shown, but as described before, parent and child events or utterances do not occur at the same time, resulting in no recurrences along the LOI. Lastly, the topology of the CRP qualitatively reveals the overall consistency of number talk across time, including when number talk occurs and recurs most often, when exchanges of number talk occur, and when speakers persist in using number utterances. Excerpts from the transcripts are also discussed below. Areas of the CRPs corresponding to those excerpts are highlighted in green boxes and number talk utterances in these excerpts are followed by an asterisk (*).

Example dyad #1: High NT frequencies, high RR. Example dyad #1 had a relatively high frequency of parent and child number talk and a high RR, and number talk utterances made up 40 percent of all utterances spoken. Both the event series and recurrence plots (Figure 2) illustrate that number talk in this interaction occurred in two clusters, one around the beginning of the transcript (between the 10th and 75th utterances) and the other toward the middle to end (between the 90th and 160th utterances). In the CRP, these "square-like" clusters are depicted along the LOI. Each cluster corresponds to segments in the transcript during which the dyad engaged in the check-out process of grocery shopping. Specifically, one partner role-played the shopper and the other played a store employee at the cash register, and they primarily discussed the prices of grocery items and the exchange of currency.

This interaction had the highest RR of the sample (7.33 percent), meaning that when one speaker used number talk, the other person reciprocated at some point in time within the interaction relatively more often than all other dyads in the sample. Also, this conversation had a DET of 33.18 percent and LAM of 34.47 percent. Thus, when number talk was being reciprocated by either speaker in this interaction, it was occurring in bouts of "back and forth" number exchanges and consecutive number utterances by either speaker, respectively, about a third of time. In the CRP, areas containing "checkerboard-like" patterns, such as the region between the 100th and 150th time points or utterances, represent times when number talk exchanges occurred. Below is an excerpt from this dyad's transcript with a relatively long number talk exchange (where each utterance was coded as number talk), at the 118th to 139th utterance (green box in the CRP in Figure 2):

- Parent (P): Okay how many is that?*
- Child (C): One, two, three, four, five, six, seven, eight.*

Figure 2 ■ Recurrence plot of example dyad #1. The tick marks along the axes indicate when the parent (blue) or child (red) spoke a number talk utterance at each time point. Transcript excerpts that are discussed in the text correspond to the green box in the CRP.

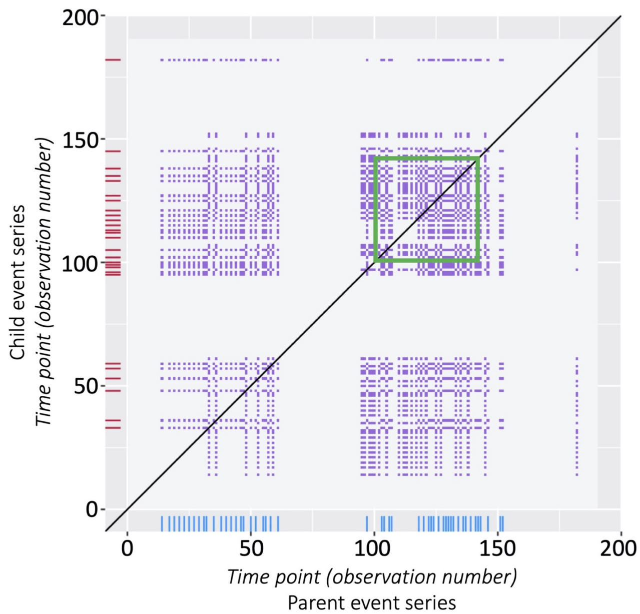
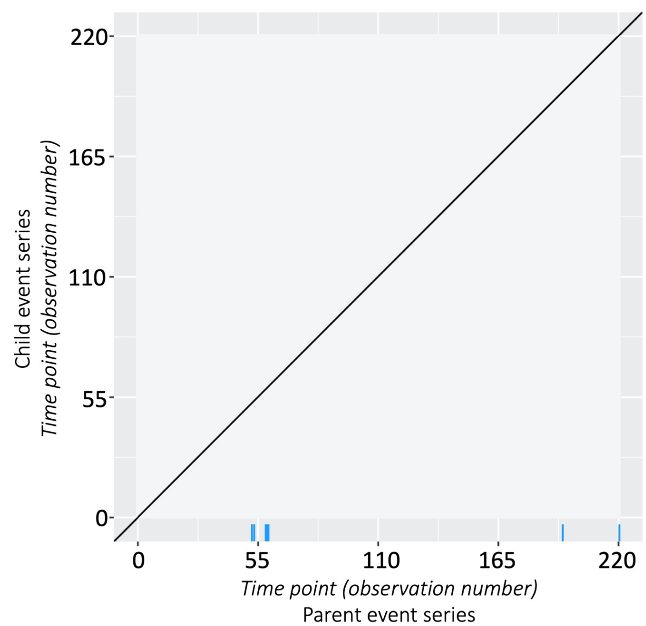


Figure 3 ■ Recurrence plot of example dyad #2.



- C: Eight, please.*
- P: Well, here's ten.*
- P: Then, we got to count up from eight.*
- C: Five.*
- P: So, do you know ten plus five?*
- C: Eight.*
- P: No, ten eleven twelve thirteen fourteen.*
- P: So, there's fifteen.*
- P: If I need to give you eighteen-*
- P: I give you fifteen.*
- P: Can you count up?*
- C: One.*
- P: Fifteen, sixteen.*
- C: Fifteen.*
- P: Eighteen.*
- P: So how many- how many dollars do I need?*
- C: Three.*
- P: Three.*

Moreover, while this dyad had a meanL of 2.38 utterances and TT of 2.44 utterances, which represent the quantity of utterances that made up number talk exchanges and consecutive number talk respectively, individual sequences of these utterances varied in length and content. For instance, in the long number talk exchange presented to the left, the parent engaged the child in counting and addition and used consecutive number utterances to scaffold this process (e.g., "Well, here's ten. Then, we got to count up from eight."). In contrast, in a shorter number talk exchange of two utterances from this dyad, such as the 145th and 146th utterances in the transcript, the child said "five" (referring to the observation that they had five dollars) and the parent simply confirmed the child's statement with "five". Thus, for this dyad, the lengths of these utterance sequences may qualitatively reflect the complexity or cognitive demand of their engagement in number talk. Overall, this interaction involved relatively high frequencies of and alignment in number talk, where many parent and child number talk utterances were closely coupled in sequences of "back and forth" exchanges or persistent number talk inputs by one speaker. Additionally, particular aspects of roleplay, such as the grocery check-out process, may have provided rich opportunities for parents and children to engage in high quality number talk, in which children could practice number skills such as counting and performing arithmetic.

Example dyad #2: Low NT frequencies, zero RR. Example dyad #2 had a relatively low frequency of parent number talk and an RR of 0. Since the child never used number talk and thus did not reciprocate the parent's number talk, there are no recurrences and the CRP (Figure 3) is blank. The transcript revealed that the parent primarily used number talk to ask the child questions about the cost of food items (e.g., the 51st utterance, "How much is my corn?") to which the child responded with a non-verbal vocalization. Later in the conversation, the parent used number talk to request permission from the child (the 201st utterance, "Can I eat one [cookie]?") and to make an observation about the quantity of cookies (the 222nd utterance, "Two cookies"), both of which did not require a number-related response from the child. An examination of the non-number utterances showed that a majority of the conversation involved identifying food items (e.g., the 135th utterance, "Is it a hamburger?") and expressing thoughts and actions (e.g., the 180th utterance, "You like that [chicken]?" and the 199th utterance, "I'm going to eat the rest"). This type of interaction, where only one person employed number talk, occurred with five dyads in the sample.

Example dyad #3: Average NT frequencies, low RR. Example dyad #3 had a relatively average frequency of parent number talk, low quantity of child number talk, and low recurrence rate. Number utterances made up 11 percent of all utterances spoken. As seen in both the event series and CRP (Figure 4), compared to example dyad #1, number talk recurred sparsely. Also, there appeared to be three segments of time during which speakers employed number talk (i.e., the very beginning, middle, and end of the conversation), and the parent tended to dominate these segments with consecutive number talk utterances. These segments corresponded to (1) the set-up of the activity during which the dyad made a couple of statements about the number of shopping baskets they were given, (2) the shopping segment during which they collected grocery items and discussed their quantities, and (3) a continuation of the shopping segment. During the periods of times when number talk did not occur or recur, the dyad identified different food items (between the 36th and 140th utterances) and discussed manipulating the food items (e.g., cutting up the apples; between 200th and 250th utterances).

Additionally, this interaction has an RR of .28 percent, suggesting that there was relatively little reciprocity in number talk between parent and child, as well as a DET of 1.45 percent and meanL of two (the minimum length of exchanges), meaning that number exchanges were rare but when they did occur, they were brief. Instead, there were more consecutive number talk utterances by one speaker, and it is clear from the CRP that this comes from the parent, which is consistent with a LAM of 26.09 percent and a TT of 3 utterances. Below are portions of this conversation from the 4th utterance, 297th to 301st utterances, and 304th to 306th utterances respectively, showing what some of these individual and consecutive utterances comprised of:

- C: [seeing the shopping baskets] We have two.*
- ...
- P: [counting while the child points to items] Yes, that's how you count.*
- P: One, two.*
- P: Three, four, five, six.*
- P: Seven.*
- P: Eight, nine, ten.*
- ...
- P: Can you count?*
- P: One, two.*
- P: Can you say the numbers?*

These excerpts highlight how number talk in this conversation tended to occur in either *individual* utterances, where one speaker was making a statement or observation that involved number, or *consecutive* number talk utterances, where one speaker (i.e., the parent) was attempting to engage the child in counting. Thus, while number talk was employed by both the parent and child, it was generally used by both speakers independently. In other words, there was little reciprocity or alignment in number talk. The following examples below (#4, #5, and #6) are intended to showcase the variability in the topology of parent-child conversations for dyads who employed average frequencies of number talk.

Example dyad #4: Average NT, average RR, high LAM. Example dyad #4 had a relatively average frequency of parent and child number talk and an average RR, and number talk utterances made up 15 percent of all utterances spoken. As shown in the event series and CRP (Figure 6), number talk occurred sparsely at the beginning of the conversation (before the 100th utterance) and then increased in frequency and density toward the end of the conversation (after the 175th utterance) in long bouts of consecutive utterances by one speaker. Moreover, this interaction had an RR of 1.06 percent, DET of 35.71 percent, and meanL of 3 utterances, meaning that when number talk was being reciprocated, it occurred in the form of "back and forth" number exchanges between the parent and child about a third of the time, and these exchanges lasted an average length of three utterances. Lastly, this interaction had a LAM of 63.69 percent, which is the largest value in the sample, and a TT of 4.76, meaning that a majority of the number talk that was reciprocated occurred in the form of consecutive number talk utterances by one speaker. In this particular interaction, both the parent and child employed several, longer-

Figure 4 ■ Recurrence plot of example dyad #3. The tick marks along the axes indicate when the parent (blue) or child (red) spoke a number talk utterance at each time point. Transcript excerpts that are discussed in the text correspond to the green box in the CRP.

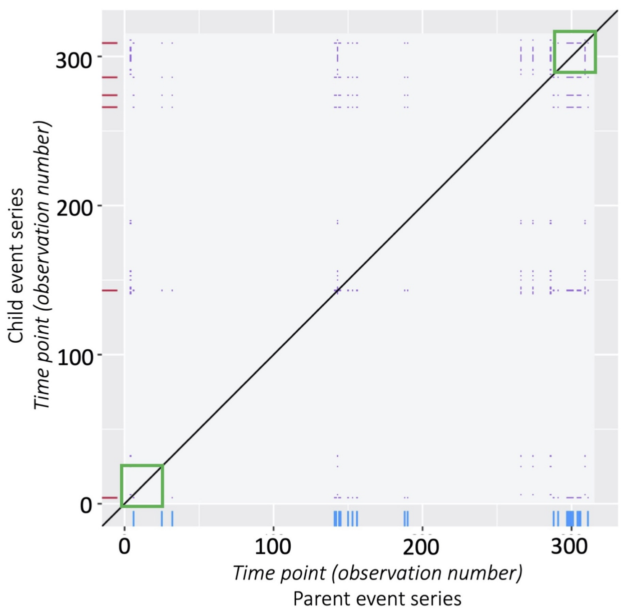
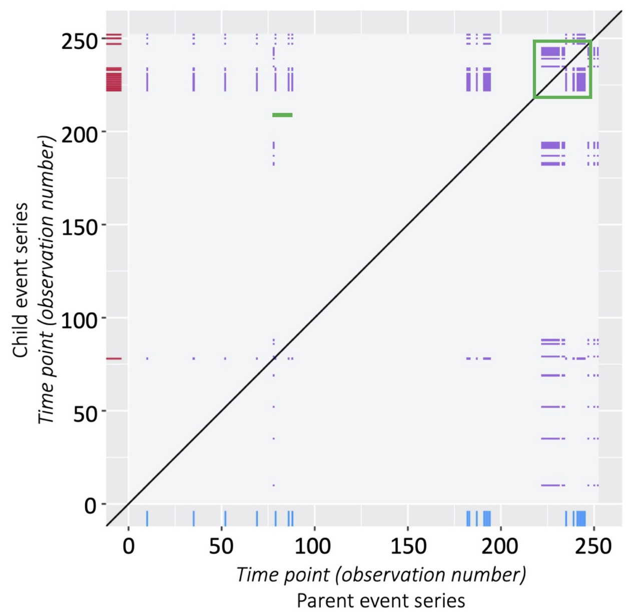


Figure 5 ■ Recurrence plot of example dyad #4. The tick marks along the axes indicate when the parent (blue) or child (red) spoke a number talk utterance at each time point. Transcript excerpts that are discussed in the text correspond to the underlined section in the event series plot and the green box in the CRP.



than-average consecutive number talk utterances toward the end of the conversation as seen in the 222nd to 231st utterances (green underlined section in the CRP) and the 241st to 245th utterances (green box in the CRP), respectively:

- C: [pressing buttons on the cash register] Nine, one, one.*
- C: One, one.*
- C: Four.*
- C: Six, six.*
- C: Eight.*
- C: Ten.*
- C: Six.*
- C: Five, five, five.*
- C: Six.*
- ...
- P: [pressing buttons on the cash register] Two.*
- P: Four.*
- P: Four, nine, three.*
- P: Four, one.*
- P: Two, one.*

Unlike example dyad #1, these long bouts of consecutive number talk utterances toward the end were the parent or child pointing to and identifying individual or sequences of numbers on the cash register. This suggests that certain features of the grocery activity, such as numbers on the toys or materials, may provide opportunities for extended bouts of number talk.

Example dyad #5: Average NT, average RR, low DET. Example dyad #5 had a relatively average frequency of parent and child number talk and average RR, and few number talk exchanges. Number utterances made up 15 percent of all utterances spoken. As seen in the event series and CRP, number talk occurred and recurred more consistently throughout the interaction compared to the previous example dyads. Like many other dyads in the sample, this parent-child pair pretended to go shopping for food and then checked out, during which they continuously discussed quantities of either food items or their prices in terms of dollars. For instance, the parent always included the number of food items they wanted to purchase, e.g., stating, “I’m going to buy two oranges” instead of saying “I’m going to buy oranges.”

Additionally, this interaction had an RR of 1.09 percent, DET of 5.18 percent, and meanL of 2 utterances. Thus, while

Figure 6 ■ Event series and recurrence plot of example dyad #5. The tick marks along the axes indicate when the parent (blue) or child (red) spoke a number talk utterance at each time point. Transcript excerpts that are discussed in the text correspond to the underlined section in the event series plot and the green box in the CRP.

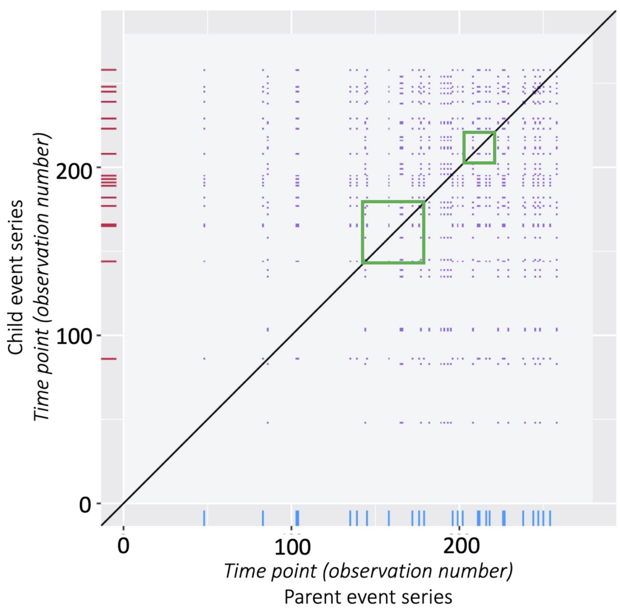
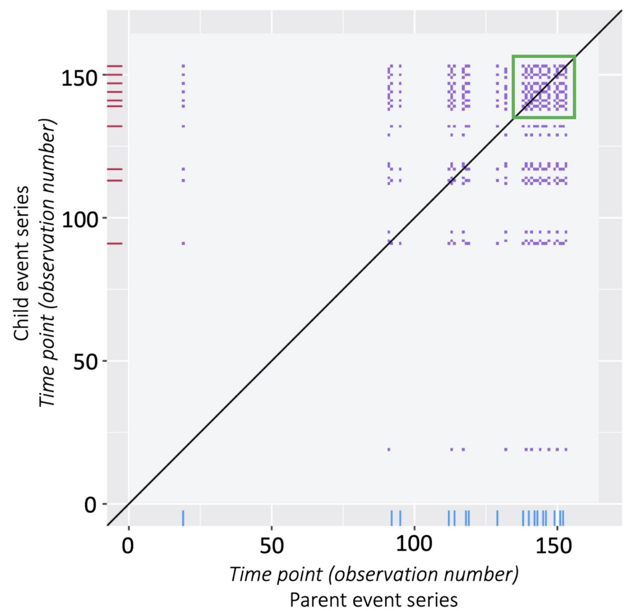


Figure 7 ■ Event series and recurrence plot of example dyad #6. The tick marks along the axes indicate when the parent (blue) or child (red) spoke a number talk utterance at each time point. Transcript excerpts that are discussed in the text correspond to the green box in the CRP.



both speakers employed number talk, it was typically not immediately reciprocated and there were few number talk exchanges. This is evident from a meanL of 2 utterances, as this is the minimum length of this metric. Below are some examples of these number talk sequences, starting with the 144th utterance and 175th utterance respectively (left green box in Figure 6):

- C: [giving a price for an item] Three dollars.*
- P: Three dollars?*
- P: Okay.
- ...
- P: Look
- P: Count how many chicken[s].*
- C: One.*
- C: Wait.
- P: Count them first.*
- P: Oh wait.
- C: Hmm.
- C: One.*
- C: [scanning food item] Beep.
- P: Oh, add it to my basket.

Thus, in this interaction, number talk exchanges generally comprised parent confirmations of their child's number talk or brief child responses to parent prompts. Further, this interaction was associated with a LAM of 17.88 percent, meaning that about one-fifth of the time that number talk was reciprocated, it resulted in persistent number talk utterances by either speaker. This is observed, for instance starting from the 208th utterance in the transcript (right green box in Figure 5):

- C: I'm going to give you five dollars.*
- P: Oh, thank you.
- C: You got blue dollar.
- P: Now I'm going to use the six dollars.*
- P: Five and one to buy six things.*

Overall, while number talk occurred and recurred more consistently throughout this conversation relative to the previous examples, because this parent attached a quantity to every food item they mentioned, it was generally not reciprocated immediately and there were fewer and shorter exchanges and consecutive number talk utterances.

Example dyad #6: Average NT, average RR, high DET. Example dyad #6 had a relatively average frequency of parent and child number talk and unlike example dyad #5, their interaction included many number talk exchanges despite featuring less number talk overall. Number talk utterances made up 16 percent of all utterances spoken. As illustrated in the event series and CRP, number talk occurred and recurred sparsely for most of the interaction and toward the end, after the 100th utterance, this dyad began to engage in many number talk exchanges. Thus, number talk showed the greatest alignment or immediate reciprocation at the end of the interaction. The occurrence and recurrence of number talk across time was consistent with the dyad's organization of the activity, such that they began by identifying and scanning food items and then shifted to paying for the food (i.e., the check-out process). The "paying" segment happened after the 100th utterance, during which the dyad briefly discussed the quantity of dollars that they had and then switched to identifying the numbers on the cash register, counting them, and performing simple arithmetic. For instance, starting from the 140th utterance, the conversation looked like this (green box in Figure 7):

- P: What is one plus one?*
- C: Two.*
- P: What is two plus two?*
- P: How many is there? *
- C: Four.*
- P: Four plus two more.*
- P: Count them.*
- C: One, two, three, four, five six.*
- P: Mhmm so what is it?
- P: Is it six?*
- C: Six.*
- P: Six plus four more.*
- P: How many is all of that together?*
- C: One, two, three, four five, six seven, eight, nine, ten.*
- P: Good job.

Moreover, this conversation had an RR of 1.26 percent, a DET of 54.71 percent, and a meanL of 2.11 utterances. In other words, this parent-child pair reciprocated each other's number talk at slightly above-average levels relative to dyads in the sample and more than half the time, this reciprocation happened in the form of number exchanges. Also, as shown in the above excerpt, these exchanges went beyond one person responding with a number talk utterance to the other speaker's number talk utterance. Similarly, this dyad had a LAM of 23.53 percent and TT of 2 utterances, meaning when number talk was reciprocated, they occurred in the form of persistent or consecutive number talk utterances, which were always a length of two sentences, by one speaker about one-fourth of the time. These consecutive number talk utterances mostly occurred when the parent prompted the child to complete a number skill, such as adding quantities, with more than one utterance (e.g., in the excerpt above, the parent asked, "Six plus four more. How many is there all together?"). Like the previous examples, this interaction shows that certain types of role play in a grocery context may have been more conducive to parent-child engagement in rich number-related conversations.

Summary of the descriptive analysis of parent-child number talk. Overall, these example dyads illustrated variability in the occurrence, recurrence, and structure of parent-child interactions surrounding number in a grocery shopping context. Across dyads, there were visible differences in the consistency and density of number utterances across time. Some dyads used number talk during specific types of role play that were more likely to elicit discussions about number concepts (e.g., the check-out process; as seen in example dyad #1), while others used number talk more consistently throughout the entire interaction (e.g., example dyad #5). In a few interactions (e.g., example dyad #2), only one speaker used number talk and thus, there was no recurrence.

Further, the extent to which dyads immediately reciprocated each other's number talk varied across dyads, even though some dyads had relatively similar quantities of number talk utterances and levels of recurrence. For instance, example dyads #3, #4, #5, and #6 all had average frequencies of parent number talk and average RRs, but they each differed in their inclusion of number talk exchanges (variability in DET), their use of consecutive number talk utterances (LAM), the length of these exchanges (meanL), and number of consecutive utterances (TT). Overall, this descriptive analysis highlights how CRQA of parent-child conversations extended beyond frequency measures of number talk, revealing differences in how dyads interacted with each other. These differences may relate to individual variability in children's early skills, a possibility that is explored in more detail below.

Research aim #2: Relations between number talk frequencies, CRQA-derived patterns of number talk, children's early skills

Pooled zero-order correlations between the parent-child number talk variables, including the frequency and CRQA measures, and child outcomes are shown in Table 5. All of the CRQA-derived parent-child interaction variables were positively correlated with each other, suggesting that dyads with greater alignment in number talk (RR) tended to have a greater occurrence of number talk exchanges (DET) and consecutive number talk utterances by one speaker (LAM) during their grocery play conversations. Further, the frequency of parent and child number talk utterances were positively correlated with all CRQA measures, with the magnitude of the correlations between parent number talk and the CRQA measures (rs range from .48 - .75) being consistently larger compared to the correlations between child number talk and the CRQA variables (rs range from .31-.71). This suggests that dyads who displayed greater alignment in number talk tended to also employ more number talk and that parents may be primarily driving these number-related discussions and eliciting number talk from the child, leading to dyadic reciprocity or alignment. Interestingly, with the exception of child number talk, none of the overall and number talk frequency variables meaningfully correlated with the child outcomes. In contrast, RR was positively related to child WJ AP (math) scores, $r(122) = .19$, $p < .05$, and none of the CRQA measures were meaningfully correlated with DVAP (vocabulary) or DCCS (EF) scores.

Extending the zero-order correlations, pooled regression models were estimated predicting each child outcome with total parent and child number talk in an initial set of models (Models 1) and then each percentage CRQA measure (Models 2) while controlling for child age. The results are shown in Table 6. Models 1 show that total number talk did not meaningfully relate to any child outcomes. Models 2 show that RR positively related to children's math skills as indexed by WJ AP scores ($B = .34$, $p < .05$) above and beyond all other recurrence measures, total number talk, and child age. None of the other variables meaningfully related to children's WJ AP scores. Also, none of the variables meaningfully related to children's expressive vocabulary as indexed by DVAP scores or their executive function skills as measured by their DCCS scores, with the exception of child age which was positively related to DCCS scores ($B = .21$ and $.20$, both ps $< .05$, in Model 1 and 2 respectively).

Discussion

The goals of this paper were to (1) demonstrate an application of cross-recurrence quantification analysis (CRQA) to derive additional measures of parent-child interactions involving number talk and (2) explore the utility of using CRQA measures to describe parent-child alignment and predict children's math skills, over and above traditional frequency measures of talk. In brief, CRQA exploits the co-occurrence of conversational events to reveal how dyads interacted with each other, such as the extent to which speakers reciprocated each other's talk. Our application of CRQA on parent-child number talk uncovered two unique structures of reciprocal utterance sequences: consecutive number talk and "back and forth" number exchanges. Thus, this method acknowledged the dynamics and adaptive nature of parent-child engagement in number-related conversations, which aligns with the potential mechanisms related to responsivity and reciprocity that may underlie the well-established association between parent-child number talk and children's early math skills (e.g., Casey et al., 2018).

First, we applied CRQA to existing transcription data of parent-child interactions during a pretend grocery shopping activity. Overall, dyadic conversations varied considerably in their alignment or reciprocity of number talk utterances (recurrence rate (RR)), the extent to which dyads engaged in number talk exchanges (determinism (DET)), the extent to which one person used consecutive number talk utterances (laminarity (LAM)), and the lengths of these exchanges (mean diagonal length (meanL)) and consecutive utterances (trapping time (TT)). Further, an in-depth descriptive analysis showed that even when dyads had similar frequencies of number talk or levels of RR, the topology of their conversations varied widely. Dyads differed in how they role-played the grocery shopping scenario, when they tended to employ number talk utterances across time, and how speakers reciprocated each other's number talk.

This application was subsequently extended by using inferential statistical methods to examine relations between CRQA-derived measures of parent-child number talk and children's early skills. While frequencies of parent and child number talk were positively associated with the CRQA-derived number talk variables, only the RR was meaningfully related to children's math skills, above and beyond total number talk, DET, LAM, and child age. Also, none of these variables, except for child age, were meaningfully linked with children's vocabulary or executive functioning skills. These findings suggest that the extent of alignment or reciprocity in parent and child number talk during play, not necessarily children's exposure to number talk, may impact and/or be shaped by children's math abilities.

The null association between the extent to which dyads engaged in number talk exchanges (DET) and children's math skills may be related to limitations of the current study, such as the relatively conservative coding scheme of the parent and child event series and the lack of specificity in the types, deliveries, and/or complexity of number talk utterances that were employed. First, future work should consider recoding the event series to capture number exchanges more liberally and examine the turn-by-turn cross-recurrence of number talk interactions. A turn refers to a unit or stretch of speech by one speaker that is comprised of at least one utterance. In conversation, speakers tend to take turns speaking at one time. The current coding scheme only coded utterances as number talk if they contained a number word that was used in a mathematical sense, but capturing number talk turns would allow utterances that

Table 4 ■ Zero-order correlations between parent-child number talk variables and child outcomes (Non-imputed data, $n = 124$)

	1	2	3	4	5	6	7	8	9	10
1. RR	–									
2. DET	0.55***	–								
3. meanL	0.39***	0.65***	–							
4. LAM	0.42***	0.46***	0.65***	–						
5. TT	0.35***	0.42***	0.77***	0.78***	–					
6. Parent utterances	−0.01	0.04	0.17	0.26**	0.24**	–				
7. Child utterances	0.00	0.04	0.10	0.09	0.03	0.49***	–			
8. Parent NT	0.75***	0.53***	0.48***	0.53***	0.48***	0.34***	0.05	–		
9. Child NT	0.71***	0.39***	0.38***	0.39***	0.31***	0.18*	0.34***	0.53***	–	
10. Total NT (Both Speakers)	0.83***	0.53***	0.50***	0.53***	0.46***	0.31***	0.21*	0.90***	0.84***	–
11. Child WJ AP	0.18*	−0.01	0.02	0.13	0.07	0.07	0.06	0.03	0.20*	0.12
12. Child DVAP	−0.06	−0.14	−0.06	0.01	−0.01	−0.02	−0.06	−0.14	0.03	−0.07
13. Child DCCS	0.03	−0.08	−0.08	−0.02	0	−0.01	−0.12	−0.05	−0.07	−0.06
14. Child age	0.13	0.10	0.19*	0.09	0.19*	−0.02	−0.09	0.09	0.09	0.10

Note. Indicators for p-values: *: $p < .05$, **: $p < .01$, ***: $p < .001$. Abbreviations: RR = Recurrence rate, DET = Determinism, meanL = Mean diagonal line length, LAM = Laminarity, TT = Trapping time, NT = Number talk, EF = Executive function, WJ AP = Woodcock-Johnson Applied Problems (child math), DVAP = Developmental Vocabulary Assessment for Parents (child vocabulary), DCCS = Dimension Change Card Sort (child executive functioning)

do not contain a number word to be coded as number talk. This is important because dyads could have discussed number concepts without using number words. This can be seen, for instance, in an exchange from example dyad #5 (utterances coded as number talk have an asterisk (*) at the end):

- P: Look
- P: Count how many chicken[s].*
- C: One.*
- C: Wait.
- P: Count them first.*
- P: Oh wait.
- C: Hmm.
- C: One.*
- C: [scanning food item] Beep.
- P: Oh, add it to my basket.

The non-number utterances in this example were involved in a number-related "back and forth" exchange, but the current coding scheme produced a conservative (i.e., shorter) estimate of the length of this exchange. Since turn-taking is a ubiquitous, naturally-occurring conversational process (e.g., Stivers et al., 2009), examining the recurrence of number talk at the level of conversational turns would increase the construct validity of number talk exchanges. Number talk turns may be a better representation of "back and forth" number exchanges as they are more closely aligned with the "natural" structure of discourse. Indeed, researchers have extended the level of analysis in CRQA to sequences of multiple utterances, i.e., turn-based recurrence, where conversational turns refer to a stretch of linguistic input from one speaker, and each turn could be coded based on the number of shared lexical, syntactic, and conceptual representations (de Wolf, 2014; Fernandez & Grimm, 2014; Grimm & Fernandez, 2014). For instance, analyzing the same corpora as Dale and Spivey (2005, 2006), Fernandez and Grimm (2014) found individual differences in all recurrence measures at these various representations. The current event series coding scheme was implemented to demonstrate that CRQA could be applied to existing data without the need to complete additional coding, which would require time and resources that researchers may not have. However, this is a limitation of the current work that may relate to the null associations between DET and children's skills, as we did not "fully" capture parent-child number exchanges. With additional coding, future work can expand the unit of analysis to examine turn-by-turn cross recurrence of number talk.

Second, the null associations between number talk exchanges (DET) and children's math skills may be related to our "global" analysis of number utterances, which did not consider the types, deliveries, and/or complexity of number talk that speakers used. Past research suggests that the complexity and delivery of conversational input matters. Specifically, parent and child engagement in "advanced" numeracy activities in the home, including conversations about large numbers or advanced concepts (e.g., cardinal-

Table 5 ■ Results of the pooled regression models predicting child outcomes with all CRQA number talk percentage variables (Imputed data, $n = 124$)

	Child outcome											
	Math skills				Vocabulary				Executive functioning			
	Model 1		Model 2		Model 1		Model 2		Model 1		Model 2	
Predictor	B	SE	B	SE	B	SE	B	SE	B	SE	B	SE
Intercept	.00	.09	.00	.08	.00	.09	.00	.09	.00	.09	.00	.09
Total NT	.12	.09	$-.14$	.18	$-.10$	.09	$-.08$	.18	$-.09$	.09	$-.30$	.17
Child age	$-.04$	.09	$-.05$	.09	.19∗	.09	.20∗	.09	.21∗	.09	.20∗	.09
RR			.34∗	.17			.04	.17			.31^	.17
DET			$-.18$	.11			$-.19$	.11			$-.14$	.11
LAM			.15	.11			.11	.11			.06	.11
Pooled R^2	.02		.07		.04		.07		.05		.08	
95% CI R^2	[.002, .09]		[.007, .17]		[.001, .14]		[.008, .18]		[.002, .15]		[.013, .19]	

Note. Indicators for p-values: *: $p < .05$, **: $p = .06$. Abbreviations: Total NT = the sum of parent and child number utterances, RR = Recurrence rate, DET = Determinism, LAM = Laminarity.

ity and arithmetic), drive the association between math-related interactions and children's math skills (e.g., Misiek et al., 2020; Ramani et al., 2015). Relatedly, the delivery of number talk differentially relates to how interactions unfold. For instance, number prompts such as directives and questions (versus non-eliciting statements) are linked with the quantity of unique number words used by children during dyadic interactions and lengthier number talk conversations (Eason et al., 2021). Additionally, while the recurrence of linguistic input tends to decrease with age and development (e.g., Dale & Spivey, 2005), this trend differs by the complexity of input. For instance, during teacher-student interactions, the recurrence of "simple" input (e.g., one-clause sentences) decreased while the recurrence of "complex" input (e.g., multiple-clause sentences) increased with time (Menninga et al., 2019). Taken together, these works suggest that different types or functions of language input can produce different conversation or discourse structures and that structures comprised of more "advanced" or "complex" input may drive the relations between dyadic interactions and children's early competencies. Therefore, future work should examine differences in the alignment or reciprocation (RR) and exchange (DET) of number talk as a function of their complexity or delivery, as these may differentially affect and be shaped by children's math development.

Additionally, future work should apply CRQA to studying the dynamics of parent-child interactions surrounding number in different informal learning contexts, such as shared book reading or cooking. Given that dyads' alignment of number talk may be important for children's early math, it may be worthwhile to examine what types of activities and their features tend to produce greater reciprocation. Relatedly, as seen in the descriptive analysis, even the topology of dyadic interactions in the same context was diverse and partially reflected differences in the ways parents and children role-played a grocery shopping scenario. Future research should extract other CRQA measures, which were previously discussed through a visual examination of the event series and CRPs, such as the consistency of recurrent events across time, entropy, or trend. Entropy refers to the consistency of diagonal line distributions on the recurrence plot. In other words, it would represent how much (dis)order there is in the lengths and occurrence of number talk exchanges. Trend refers to the extent to which the density of recurrent points varies as they move away from the line of incidence. Qualitatively, this would represent whether dyads tended to immediately reciprocate each other's number talk.

In sum, this paper extended existing frequency measures of parent-child interactions, particularly surrounding numbers and math, using CRQA, an analytic method that acknowledges the dynamics, adaptivity, and richness of dyadic conversations. This application revealed that the extent to which parents and children reciprocated each other's number talk utterances was linked with children's early math, but not vocabulary or executive functioning skills. Broadly speaking, the reciprocal and interactive nature of parent-child number talk may reflect a domain-specific social process that supports children's early math. This work has practical implications for the design and encouragement of informal learning activities that leverage parents' and children's existing conversational practices surrounding number and math.

Authors' note

The data and tutorial code are available as supplementary material and can be accessed at github.com/s-duong/crqa-

number-talk. The first author can also be contacted at shirleyduong5@gmail.com.

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Open practices

The *Open Data* badge was earned because the data of the experiment(s) are available on github.com/s-duong/crqa-number-talk

The *Open Material* badge was earned because supplementary material(s) are available on the journal's web site.

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